

### 5.3 Three-Stage Nested Design Example

A researcher designs an experiment to study the hardness of a metal alloy. A three-stage nested design was conducted that included

- Two alloy chemistry compositions.
- Three ovens for each alloy chemistry composition (6 ovens were used).
- Four ingot molds were used to produce alloy ingots for each of the six combinations of alloy chemistry composition and oven (24 molds were used).
- Three ingots were produced from each of the 24 molds. Molds can only be used once.

The experimental data in the table below contains alloy hardness measurements.

		Alloy Chemistry											
		1				2				3			
Oven		1				2				3			
Mold		1	2	3	4	1	2	3	4	1	2	3	4
		42.5	43.1	37.0	50.7	61.6	58.8	61.9	53.9	58.0	53.8	59.5	55.9
		46.5	48.1	39.0	53.7	59.6	62.8	60.9	59.9	59.0	50.8	57.5	46.9
		44.5	40.1	43.0	47.7	61.6	57.8	52.9	57.9	61.0	53.8	55.5	51.9

		Alloy Chemistry											
		1				2				3			
Oven		1				2				3			
Mold		1	2	3	4	1	2	3	4	1	2	3	4
		39.5	37.8	45.0	37.8	59.0	63.9	63.8	58.0	56.7	50.8	52.7	45.7
		35.5	38.8	38.0	38.8	59.0	61.9	65.8	60.0	50.7	50.8	56.7	47.7
		37.5	41.8	42.0	41.8	60.0	59.9	59.8	62.0	52.7	58.8	57.7	49.7

Analyze this data assuming **alloy chemistry and ovens are fixed effects** and **ingot mold is a random effect**. Assume data collection was randomized.

- Based on the expected mean squares, the  $F$ -statistics are

$$F_{\text{alloy}} = \frac{MS_{\text{alloy}}}{MS_{\text{mold}(\text{alloy}*\text{oven})}} \quad F_{\text{oven}(\text{alloy})} = \frac{MS_{\text{oven}(\text{alloy})}}{MS_{\text{mold}(\text{alloy}*\text{oven})}} \quad F_{\text{mold}(\text{oven}*\text{alloy})} = \frac{MS_{\text{mold}(\text{oven}*\text{alloy})}}{MS_E}$$

- The tests for  $\text{oven}(\text{alloy})$  and  $\text{mold}(\text{oven}*\text{alloy})$  are significant with  $p$ -values of  $< .0001$  and  $.0004$ , respectively.
- The variance component estimates are  $\hat{\sigma}^2 =$                       and  $\hat{\sigma}_{\text{mold}}^2 =$                       . Thus, the random replication variability is estimated to be about 27% larger than the variability due to molds.

#### **THREE-STAGE NESTED DESIGN: ALLOY AND oven FIXED , MOLD RANDOM**

##### **The GLM Procedure**

Class Level Information		
Class	Levels	Values
alloy	2	1 2
oven	3	1 2 3
mold	4	1 2 3 4

**Variable: hardness**

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	23	4742.804444	206.208889	25.42	<.0001
Error	48	389.333333	8.111111		
Corrected Total	71	5132.137778			

R-Square	Coeff Var	Root MSE	hardness Mean
0.924138	5.472248	2.848001	52.04444

Source	DF	Type III SS	Mean Square	F Value	Pr > F
alloy	1	70.013889	70.013889	8.63	0.0051
oven(alloy)	4	4181.087222	1045.271806	128.87	<.0001
mold(alloy*oven)	18	491.703333	27.316852	3.37	0.0004

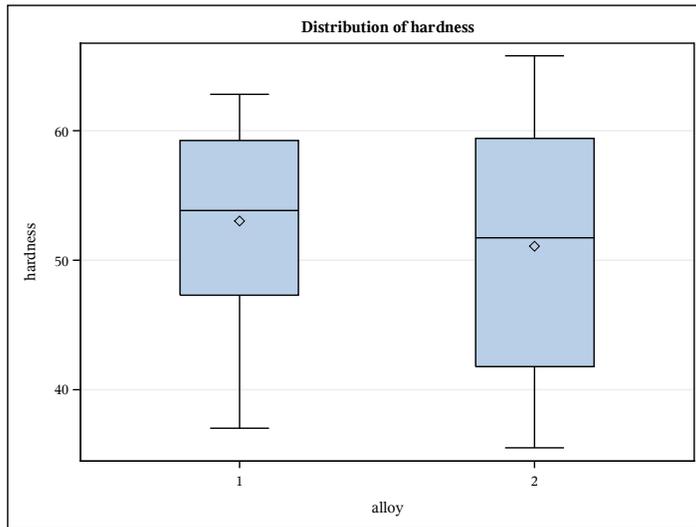
Source	Type III Expected Mean Square
alloy	Var(Error) + 3 Var(mold(alloy*oven)) + Q(alloy,oven(alloy))
oven(alloy)	Var(Error) + 3 Var(mold(alloy*oven)) + Q(oven(alloy))
mold(alloy*oven)	Var(Error) + 3 Var(mold(alloy*oven))

**Tests of Hypotheses for Mixed Model Analysis of Variance**

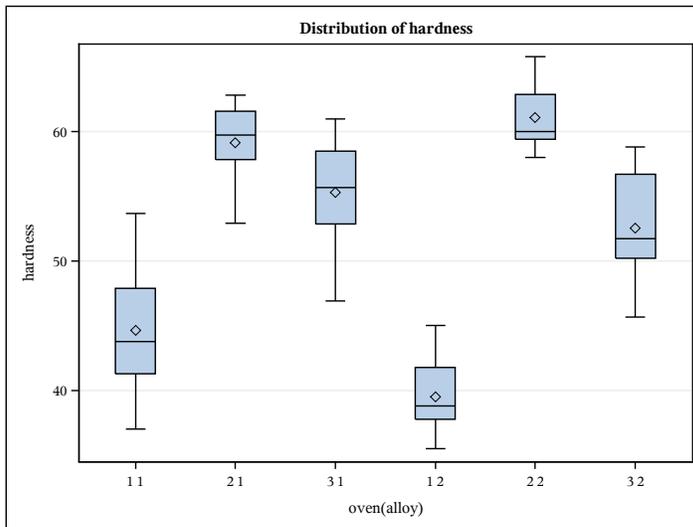
**Variable: hardness**

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	alloy	1	70.013889	70.013889	2.56	0.1268
	oven(alloy)	4	4181.087222	1045.271806	38.26	<.0001
	Error	18	491.703333	27.316852		
Error: MS(mold(alloy*oven))						
* This test assumes one or more other fixed effects are zero.						

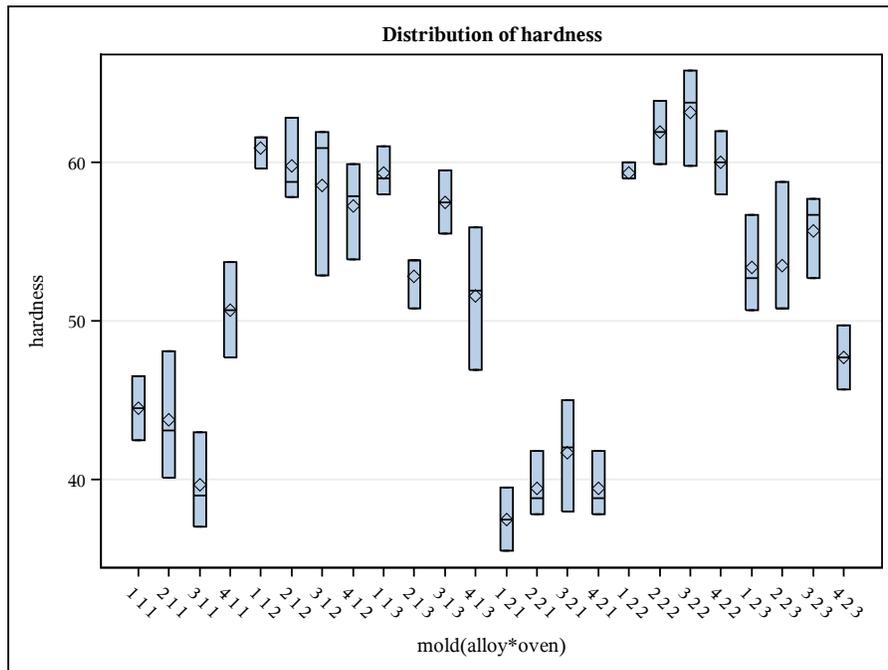
Source	DF	Type III SS	Mean Square	F Value	Pr > F
mold(alloy*oven)	18	491.703333	27.316852	3.37	0.0004
Error: MS(Error)	48	389.333333	8.111111		



Level of alloy	N	hardness	
		Mean	Std Dev
1	36	53.0305556	7.37558983
2	36	51.0583333	9.49909394



Level of oven	Level of alloy	N	hardness	
			Mean	Std Dev
1	1	12	44.6583333	4.89943565
2	1	12	59.1333333	3.11662210
3	1	12	55.3000000	4.08122085
1	2	12	39.5250000	2.62994124
2	2	12	61.0916667	2.38649927
3	2	12	52.5583333	4.13619464



Level of mold	Level of alloy	Level of oven	N	hardness	
				Mean	Std Dev
1	1	1	3	44.5000000	2.0000000
2	1	1	3	43.7666667	4.04145188
3	1	1	3	39.6666667	3.05505046
4	1	1	3	50.7000000	3.0000000
1	1	2	3	60.9333333	1.15470054
2	1	2	3	59.8000000	2.64575131
3	1	2	3	58.5666667	4.93288286
4	1	2	3	57.2333333	3.05505046
1	1	3	3	59.3333333	1.52752523
2	1	3	3	52.8000000	1.73205081
3	1	3	3	57.5000000	2.0000000
4	1	3	3	51.5666667	4.50924975
1	2	1	3	37.5000000	2.0000000
2	2	1	3	39.4666667	2.08166600
3	2	1	3	41.6666667	3.51188458

Level of mold	Level of alloy	Level of oven	N	hardness	
				Mean	Std Dev
4	2	1	3	39.4666667	2.08166600
1	2	2	3	59.3333333	0.57735027
2	2	2	3	61.9000000	2.0000000
3	2	2	3	63.1333333	3.05505046
4	2	2	3	60.0000000	2.0000000
1	2	3	3	53.3666667	3.05505046
2	2	3	3	53.4666667	4.61880215
3	2	3	3	55.7000000	2.64575131
4	2	3	3	47.7000000	2.0000000

## VARIANCE COMPONENTS ANALYSIS

### Variance Components Estimation Procedure

REML Estimates	
Variance Component	Estimate
Var(mold(alloy*oven))	6.40191
Var(Error)	8.11111

Analyze the data again but assume **chemistry alloy is a fixed effect** and **oven and mold are random effects**. That is, the 6 ovens were randomly selected from a larger population of ovens.

- Based on the expected mean squares, the  $F$ -statistics are

$$F_{alloy} = \frac{MS_{alloy}}{MS_{oven(alloy)}} \qquad F_{oven(alloy)} = \frac{MS_{oven(alloy)}}{MS_{mold(alloy*oven)}}$$

$$F_{mold(oven*alloy)} = \frac{MS_{mold(oven*alloy)}}{MSE}$$

- The tests for variance components for  $oven(alloy)$  and  $mold(oven*alloy)$  are significant with  $p$ -values of  $< .0001$  and  $.0004$ , respectively.
- The variance component estimates are  $\hat{\sigma}^2 =$  ,  $\hat{\sigma}_{mold}^2 =$  , and  $\hat{\sigma}_{oven}^2 =$
- This implies there is very large variability due to changes in oven. It is estimated to be about 10.5 times larger than the random replication variability, and it is about 13.3 times larger than the variability due to molds. The engineers need to first focus on reducing the oven-to-oven variability.

**THREE-STAGE NESTED DESIGN: ALLOY FIXED, oven AND MOLD RANDO**

**The GLM Procedure**

**Variable: hardness**

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	23	4742.804444	206.208889	25.42	<.0001
Error	48	389.333333	8.111111		
Corrected Total	71	5132.137778			

R-Square	Coeff Var	Root MSE	hardness Mean
0.924138	5.472248	2.848001	52.04444

Source	DF	Type III SS	Mean Square	F Value	Pr > F
alloy	1	70.013889	70.013889	8.63	0.0051
oven(alloy)	4	4181.087222	1045.271806	128.87	<.0001
mold(alloy*oven)	18	491.703333	27.316852	3.37	0.0004

Source	Type III Expected Mean Square
alloy	Var(Error) + 3 Var(mold(alloy*oven)) + 12 Var(oven(alloy)) + Q(alloy)
oven(alloy)	Var(Error) + 3 Var(mold(alloy*oven)) + 12 Var(oven(alloy))
mold(alloy*oven)	Var(Error) + 3 Var(mold(alloy*oven))

**The GLM Procedure**  
**Tests of Hypotheses for Mixed Model Analysis of Variance**

**Variable: hardness**

Source	DF	Type III SS	Mean Square	F Value	Pr > F
alloy	1	70.013889	70.013889	0.07	0.8086
Error	4	4181.087222	1045.271806		
<b>Error: MS(oven(alloy))</b>					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
oven(alloy)	4	4181.087222	1045.271806	38.26	<.0001
Error	18	491.703333	27.316852		
<b>Error: MS(mold(alloy*oven))</b>					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
mold(alloy*oven)	18	491.703333	27.316852	3.37	0.0004
Error: MS(Error)	48	389.333333	8.111111		

REML Iterations				
Iteration	Objective	Var(oven(alloy))	Var(mold(alloy*oven))	Var(Error)
0	187.8184707119	84.8295794753	6.4019135802	8.1111111111
1	187.8184707119	84.8295794753	6.4019135802	8.1111111111

Convergence criteria met.

REML Estimates	
Variance Component	Estimate
Var(oven(alloy))	84.82958
Var(mold(alloy*oven))	6.40191
Var(Error)	8.11111

## SAS Code for Three-Stage Nested Design Example

```
DM 'LOG; CLEAR; OUT; CLEAR;';

ODS GRAPHICS ON;
ODS PRINTER PDF file='C:\COURSES\ST541\NESTED3.PDF';
OPTIONS NODATE NONUMBER;

*****;
*** THREE-STAGE NESTED DESIGNS ***;
*****;
DATA in;
  DO alloy=1 TO 2;
    DO oven=1 TO 3;
      DO mold=1 TO 4;
        DO ingot = 1 to 3;
          INPUT hardness @@; OUTPUT;
        END; END; END; END;
LINES;
42.5 46.5 44.5 43.1 48.1 40.1 37.0 39.0 43.0 50.7 53.7 47.7
61.6 59.6 61.6 58.8 62.8 57.8 61.9 60.9 52.9 53.9 59.9 57.9
58.0 59.0 61.0 53.8 50.8 53.8 59.5 57.5 55.5 55.9 46.9 51.9
39.5 35.5 37.5 37.8 38.8 41.8 45.0 38.0 42.0 37.8 38.8 41.8
59.0 59.0 60.0 63.9 61.9 59.9 63.8 65.8 59.8 58.0 60.0 62.0
56.7 50.7 52.7 50.8 50.8 58.8 52.7 56.7 57.7 45.7 47.7 49.7
;

PROC GLM DATA=in;
  CLASS alloy oven mold;
  MODEL hardness = alloy oven(alloy) mold(alloy oven) / SS3;
  RANDOM mold(alloy oven) / TEST;
  MEANS alloy oven(alloy) mold(alloy oven);
TITLE 'THREE-STAGE NESTED DESIGN: ALLOY and OVEN FIXED , MOLD RANDOM';

PROC VARCOMP DATA=in METHOD=REML;
  CLASS alloy oven mold;
  MODEL hardness = alloy oven(alloy) mold(alloy oven) / FIXED=2;
TITLE 'VARIANCE COMPONENTS ANALYSIS';

PROC GLM DATA=in;
  CLASS alloy oven mold;
  MODEL hardness = alloy oven(alloy) mold(alloy oven) / SS3;
  RANDOM oven(alloy) mold(alloy oven) / TEST;
  * MEANS alloy oven(alloy) mold(alloy oven);
TITLE 'THREE-STAGE NESTED DESIGN: ALLOY FIXED, OVEN and MOLD RANDOM';

PROC VARCOMP DATA=in METHOD=REML;
  CLASS alloy oven mold;
  MODEL hardness = alloy oven(alloy) mold(alloy oven) / FIXED=1;
TITLE 'VARIANCE COMPONENTS ANALYSIS';
RUN;
```

## 5.4 Examples with Crossed and Nested Effects

### 5.4.1 Three Factor Example

An industrial engineer is studying the manual assembly of electronic components on circuit boards. The goal is to improve the speed of the assembly operation.

- The engineer has designed **three assembly fixtures** and **two workplace layouts** that seem promising.
- Operators are required to manually perform the assembly, and it is was decided to randomly select **four operators** for each fixture and layout combination.
- Because the workplaces are in different locations within the plant, it is difficult to use the same four operators for each layout. Therefore, the four operators chosen for layout 1 are different individuals than the four operators chosen for layout 2.
- Because the eight operators were also chosen at random, we have a mixed model containing both crossed (factorial) and nested effects.
- The data collection was completely randomized and **two replicates** were obtained. The assembly times measured in seconds are given in the table below.

Operator	Layout 1				Layout 2				$y_{i...}$
	1	2	3	4	1	2	3	4	
Fixture 1	22	23	28	25	26	27	28	24	404
	24	24	29	23	28	25	25	24	
Fixture 2	30	29	30	27	29	30	24	28	447
	27	28	32	25	28	27	23	30	
Fixture 3	25	24	27	26	27	26	24	28	401
	21	22	25	23	25	24	27	27	
$y_{.jk}$	149	150	171	149	163	159	151	160	
$y_{.j.}$	619				633				$y_{....} = 1252$

What is the model equation associated with this design?

## **DESIGN WITH NESTED AND CROSSED FACTORS**

### **The GLM Procedure**

Class Level Information		
Class	Levels	Values
fixture	3	1 2 3
layout	2	1 2
operator	4	1 2 3 4

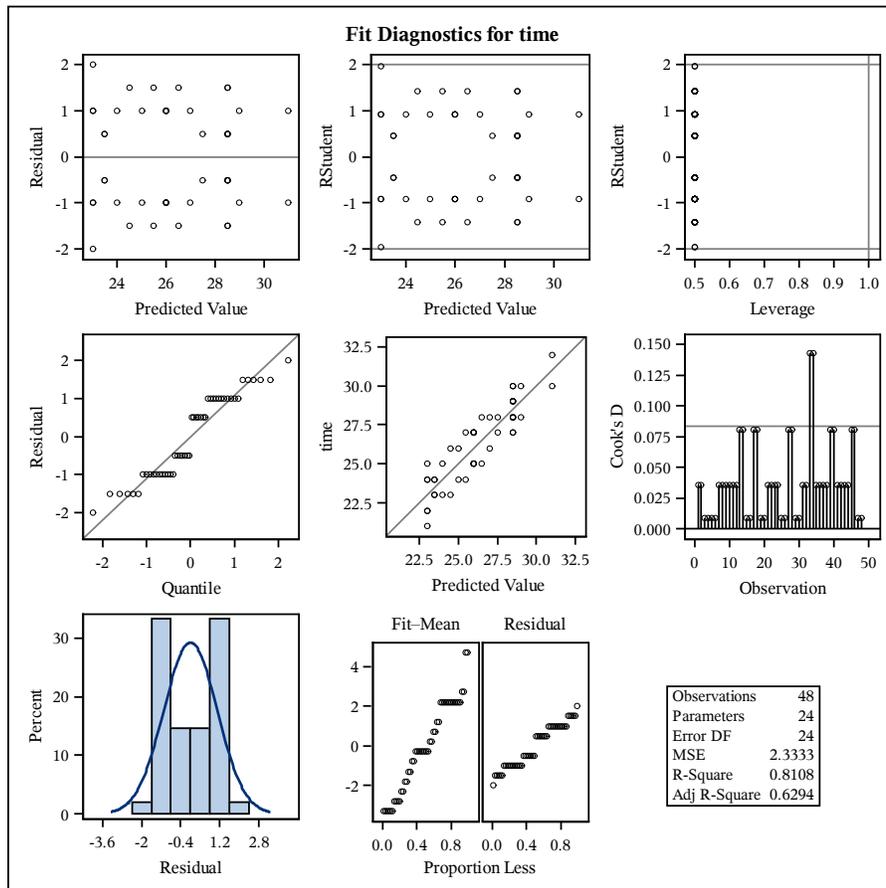
Number of Observations Read	48
Number of Observations Used	48

**Variable: time**

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	23	239.9166667	10.4311594	4.47	0.0003
Error	24	56.0000000	2.3333333		
Corrected Total	47	295.9166667			

R-Square	Coeff Var	Root MSE	time Mean
0.810758	5.809922	1.527525	26.29167

Source	DF	Type III SS	Mean Square	F Value	Pr > F
fixture	2	124.0416667	62.0208333	26.58	<.0001
layout	1	12.0000000	12.0000000	5.14	0.0326
fixture*layout	2	6.1250000	3.0625000	1.31	0.2878
operator(layout)	6	60.2500000	10.0416667	4.30	0.0044
fixtu*operat(layout)	12	37.5000000	3.1250000	1.34	0.2610



Source	Type III Expected Mean Square
fixture	$\text{Var}(\text{Error}) + 2 \text{Var}(\text{fixtu}*\text{operat}(\text{layout})) + Q(\text{fixture}, \text{fixture}*\text{layout})$
layout	$\text{Var}(\text{Error}) + 2 \text{Var}(\text{fixtu}*\text{operat}(\text{layout})) + 6 \text{Var}(\text{operator}(\text{layout})) + Q(\text{layout}, \text{fixture}*\text{layout})$
fixture*layout	$\text{Var}(\text{Error}) + 2 \text{Var}(\text{fixtu}*\text{operat}(\text{layout})) + Q(\text{fixture}*\text{layout})$
operator(layout)	$\text{Var}(\text{Error}) + 2 \text{Var}(\text{fixtu}*\text{operat}(\text{layout})) + 6 \text{Var}(\text{operator}(\text{layout}))$
fixtu*operat(layout)	$\text{Var}(\text{Error}) + 2 \text{Var}(\text{fixtu}*\text{operat}(\text{layout}))$

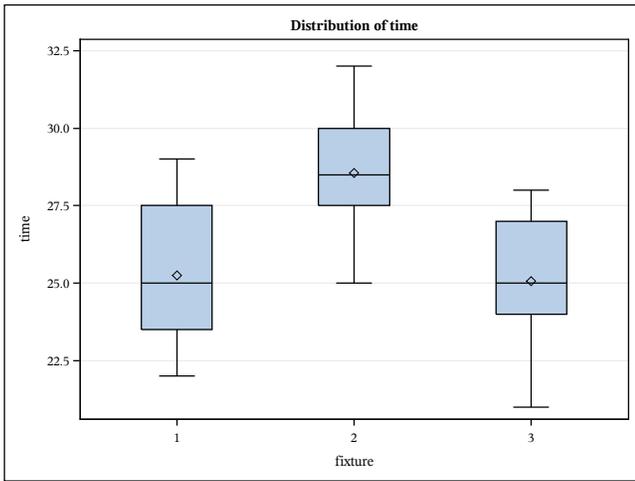
***The GLM Procedure***  
***Tests of Hypotheses for Mixed Model Analysis of Variance***

**Variable: time**

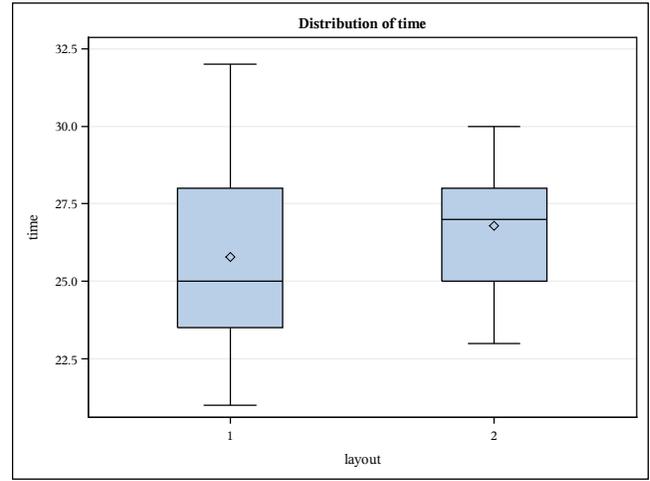
	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	fixture	2	124.041667	62.020833	19.85	0.0002
	fixture*layout	2	6.125000	3.062500	0.98	0.4034
	operator(layout)	6	60.250000	10.041667	3.21	0.0404
	Error	12	37.500000	3.125000		
<b>Error: MS(fixtu*operat(layout))</b>						
<b>* This test assumes one or more other fixed effects are zero.</b>						

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	layout	1	12.000000	12.000000	1.20	0.3163
	Error	6	60.250000	10.041667		
<b>Error: MS(operator(layout))</b>						
<b>* This test assumes one or more other fixed effects are zero.</b>						

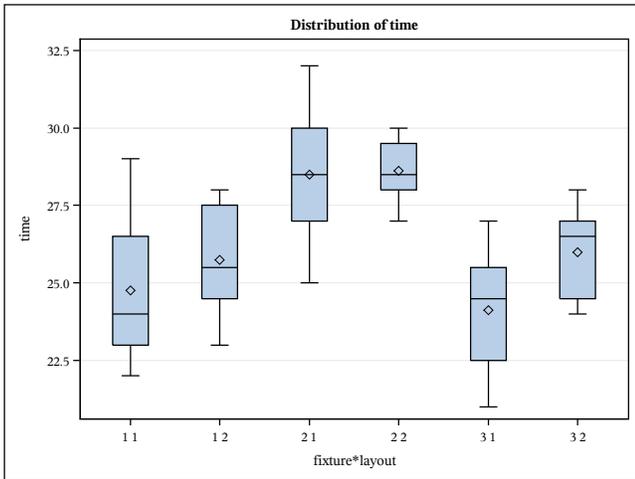
Source	DF	Type III SS	Mean Square	F Value	Pr > F
fixtu*operat(layout)	12	37.500000	3.125000	1.34	0.2610
Error: MS(Error)	24	56.000000	2.333333		



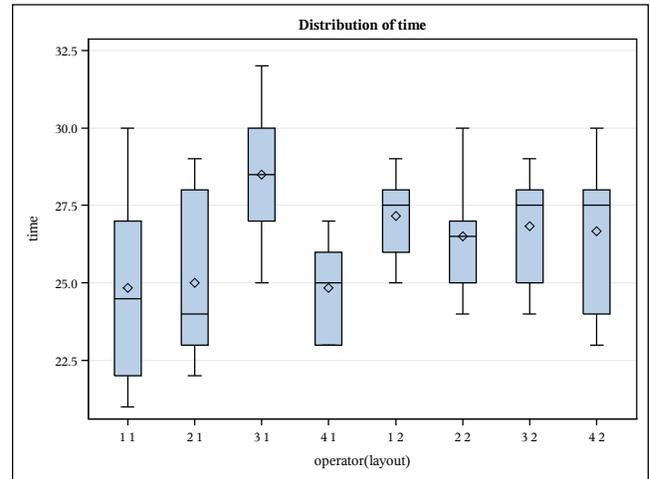
Level of fixture	N	time	
		Mean	Std Dev
1	16	25.2500000	2.17562252
2	16	28.5625000	1.67207456
3	16	25.0625000	1.98221257



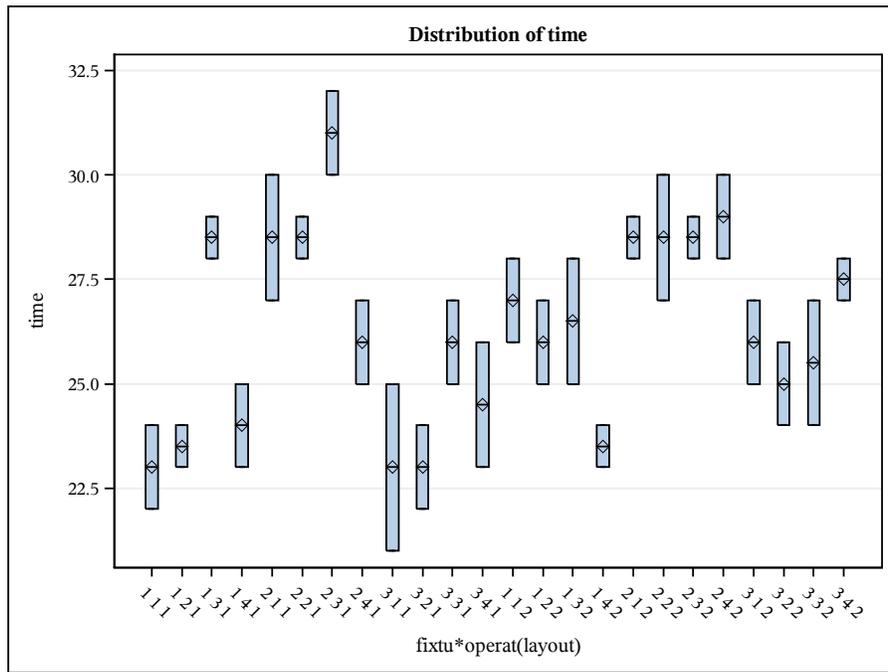
Level of layout	N	time	
		Mean	Std Dev
1	24	25.7916667	2.91889149
2	24	26.7916667	1.95557546



Level of fixture	Level of layout	N	time	
			Mean	Std Dev
1	1	8	24.7500000	2.49284691
1	2	8	25.7500000	1.83225076
2	1	8	28.5000000	2.20389266
2	2	8	28.6250000	1.06066017
3	1	8	24.1250000	2.03100960
3	2	8	26.0000000	1.51185789



Level of operator	Level of layout	N	time	
			Mean	Std Dev
1	1	6	24.8333333	3.31159579
2	1	6	25.0000000	2.82842712
3	1	6	28.5000000	2.42899156
4	1	6	24.8333333	1.60208198
1	2	6	27.1666667	1.47196014
2	2	6	26.5000000	2.07364414
3	2	6	26.8333333	1.94079022
4	2	6	26.6666667	2.65832027



Level of fixture	Level of operator	Level of layout	N	time	
				Mean	Std Dev
1	1	1	2	23.0000000	1.41421356
1	2	1	2	23.5000000	0.70710678
1	3	1	2	28.5000000	0.70710678
1	4	1	2	24.0000000	1.41421356
2	1	1	2	28.5000000	2.12132034
2	2	1	2	28.5000000	0.70710678
2	3	1	2	31.0000000	1.41421356
2	4	1	2	26.0000000	1.41421356
3	1	1	2	23.0000000	2.82842712
3	2	1	2	23.0000000	1.41421356
3	3	1	2	26.0000000	1.41421356
3	4	1	2	24.5000000	2.12132034
1	1	2	2	27.0000000	1.41421356
1	2	2	2	26.0000000	1.41421356
1	3	2	2	26.5000000	2.12132034

Level of fixture	Level of operator	Level of layout	N	time	
				Mean	Std Dev
1	4	2	2	23.5000000	0.70710678
2	1	2	2	28.5000000	0.70710678
2	2	2	2	28.5000000	2.12132034
2	3	2	2	28.5000000	0.70710678
2	4	2	2	29.0000000	1.41421356
3	1	2	2	26.0000000	1.41421356
3	2	2	2	25.0000000	1.41421356
3	3	2	2	25.5000000	2.12132034
3	4	2	2	27.5000000	0.70710678

Tests for Normality				
Test	Statistic		p Value	
Shapiro-Wilk	W	0.906905	Pr < W	0.0011
Kolmogorov-Smirnov	D	0.176546	Pr > D	<0.0100
Cramer-von Mises	W-Sq	0.361631	Pr > W-Sq	<0.0050
Anderson-Darling	A-Sq	2.003272	Pr > A-Sq	<0.0050

SAS Code for Three Factor Example

```

DM 'LOG; CLEAR; OUT; CLEAR;';
ODS GRAPHICS ON;
ODS PRINTER PDF file='C:\COURSES\ST541\NESTWF.PDF';
OPTIONS NODATE NONUMBER;

*****
*** NESTED AND CROSSED FACTORS ***
*****
DATA in;
  DO fixture=1 TO 3;
    DO layout=1 TO 2;
      DO operator=1 TO 4;
        DO rep=1 TO 2;
          INPUT time @@; OUTPUT;
        END; END; END; END;
LINES;
22 24 23 24 28 29 25 23 26 28 27 25 28 25 24 23
30 27 29 28 30 32 27 25 29 28 30 27 28 29 28 30
25 21 24 22 27 25 26 23 27 25 26 24 24 27 28 27
;
PROC GLM DATA=in PLOTS=(ALL);
  CLASS fixture layout operator;
  MODEL time = fixture|layout fixture|operator(layout) / SS3;
  RANDOM operator(layout) fixture*operator(layout) / TEST;
  MEANS fixture|layout fixture|operator(layout) ;
  OUTPUT OUT=diag R=resid;
TITLE 'DESIGN WITH NESTED AND CROSSED FACTORS';

PROC UNIVARIATE DATA= diag NORMAL;
  VAR resid;
RUN;

```

Partitioning the  $SS_{total}$

$$\begin{aligned}
 SS_{TOTAL} &= \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^c \sum_{l=1}^n (y_{ijkl} - \bar{y}_{....})^2 && a=3 \quad c=4 \\
 &&& b=2 \quad n=2 \\
 &= \sum_i \sum_j \sum_k \sum_l \left[ \overset{\tau_i}{(\bar{y}_{i...} - \bar{y}_{....})^2} + \overset{\beta_j}{(\bar{y}_{.j..} - \bar{y}_{....})^2} \right. \\
 &\quad + \overset{(\tau\beta)_{ij}}{(\bar{y}_{ij..} - \bar{y}_{i...} - \bar{y}_{.j..} + \bar{y}_{....})^2} + \overset{\delta_{kl}}{(\bar{y}_{.j.k.} - \bar{y}_{.j..})^2} \\
 &\quad \left. + \overset{(\tau\gamma)_{ikl}}{(\bar{y}_{ij.k.} - \bar{y}_{ij..} - \bar{y}_{.j.k.} + \bar{y}_{.j..})^2} + \overset{\epsilon_{ijkl}}{(y_{ijkl} - \bar{y}_{ij.k.})^2} \right]^2 \\
 &= bcn \sum_i (\bar{y}_{i...} - \bar{y}_{....})^2 + acn \sum_j (\bar{y}_{.j..} - \bar{y}_{....})^2 + cn \sum_i \sum_j (\bar{y}_{ij..} - \bar{y}_{i...} - \bar{y}_{.j..} + \bar{y}_{....})^2 \\
 &\quad + an \sum_j \sum_k (\bar{y}_{.j.k.} - \bar{y}_{.j..})^2 + n \sum_i \sum_j \sum_k (\bar{y}_{ij.k.} - \bar{y}_{ij..} - \bar{y}_{.j.k.} + \bar{y}_{.j..})^2 + \sum_i \sum_j \sum_k \sum_l (y_{ijkl} - \bar{y}_{ij.k.})^2 \\
 &\quad + 2 \sum_i \sum_j \sum_k \sum_l \text{(CROSS PRODUCTS)} \\
 &= SS_A + SS_B + SS_{AB} + SS_{C(B)} + SS_{A-C(B)} + SS_E
 \end{aligned}$$

### 5.4.2 Four Factor Example

For the following quality control example, Suppliers and Operators are **fixed** effects. The two production runs for operator D are different than the two production runs for operator N. From each supplier, two random batches of raw materials were used in this study. Hence, Production Run and Batches are **random** effects.

		Supplier					
		A		B		C	
		Batch		Batch		Batch	
		3	6	1	4	2	5
Operator D	Run 1	21.21	23.61	15.16	12.24	24.20	22.83
		21.32	24.82	16.41	13.18	23.99	22.75
		19.95	24.84	15.47	12.83	23.40	22.46
	Run 2	25.77	25.24	19.07	15.07	25.75	26.23
		26.15	25.22	17.44	15.14	25.12	25.11
		26.15	25.02	18.16	15.07	25.08	25.81
Operator N	Run 3	27.89	33.04	23.22	21.58	32.40	30.54
		27.89	31.98	22.51	21.11	31.52	30.67
		28.28	31.76	24.67	20.96	31.93	30.34
	Run 4	29.13	32.93	23.10	22.14	30.58	33.39
		28.50	33.85	22.71	21.72	30.26	33.63
		28.68	34.22	22.94	22.11	31.02	32.69

What is the model equation associated with this design?

- There are **exact**  $F$ -tests for Run(Operator), Batch(Supplier), Supplier\*Run(Operator), Operator\*Batch(Supplier), and Run(Operator)\*Batch(Supplier).
- There are **approximate**  $F$ -tests for Operator, Supplier, and Operator\*Supplier.
- The REML (restricted maximum likelihood) estimates of the variance components were generated. REML estimates do not allow for negative variance components estimates. That is why the one of the estimates is set to 0.
- The Supplier\*Operator interaction plots indicate parallel lines which is consistent with the  $p$ -value of .8867.

### **QUALITY CONTROL EXAMPLE-- 2 CROSSED AND 2 NESTED EFFECTS**

#### **The GLM Procedure**

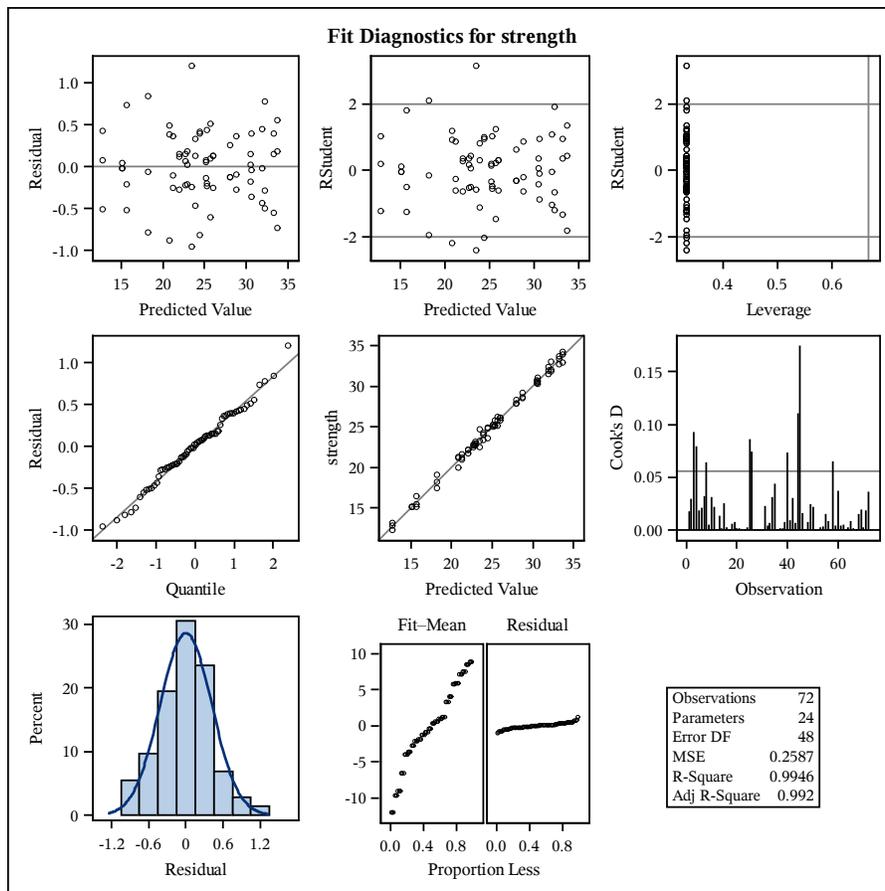
Class Level Information		
Class	Levels	Values
operator	2	D N
supplier	3	A B C
run	4	1 2 3 4
batch	6	1 2 3 4 5 6

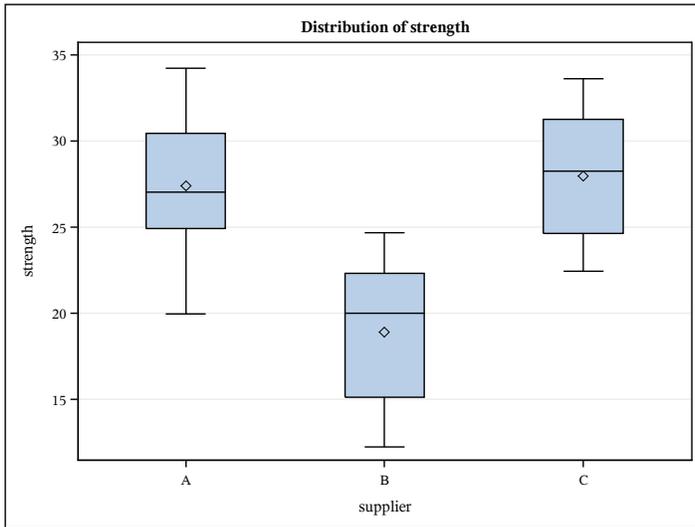
**Variable: strength**

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
<b>Model</b>	23	2295.503778	99.804512	385.75	<.0001
<b>Error</b>	48	12.418933	0.258728		
<b>Corrected Total</b>	71	2307.922711			

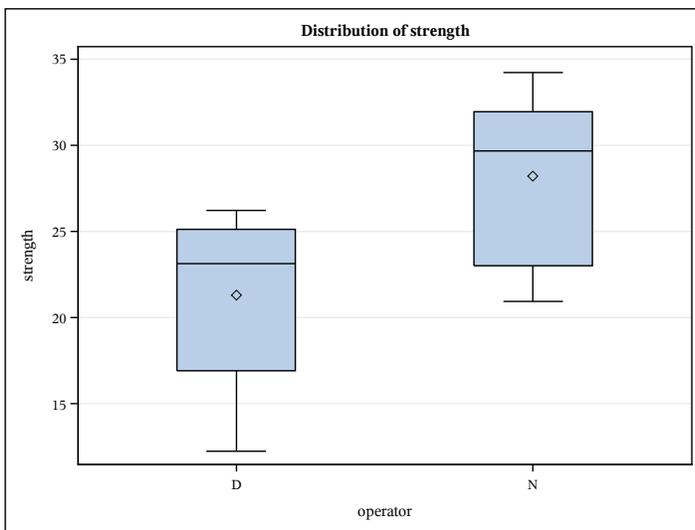
R-Square	Coeff Var	Root MSE	strength Mean
0.994619	2.053826	0.508653	24.76611

Source	DF	Type III SS	Mean Square	F Value	Pr > F
<b>operator</b>	1	858.498672	858.498672	3318.15	<.0001
<b>supplier</b>	2	1235.831003	617.915501	2388.28	<.0001
<b>operator*supplier</b>	2	1.164703	0.582351	2.25	0.1163
<b>run(operator)</b>	2	62.152250	31.076125	120.11	<.0001
<b>batch(supplier)</b>	3	84.889842	28.296614	109.37	<.0001
<b>supplie*run(operator)</b>	4	2.255683	0.563921	2.18	0.0854
<b>operat*batch(suppli)</b>	3	19.944025	6.648008	25.69	<.0001
<b>run*batc(oper*suppl)</b>	6	30.767600	5.127933	19.82	<.0001

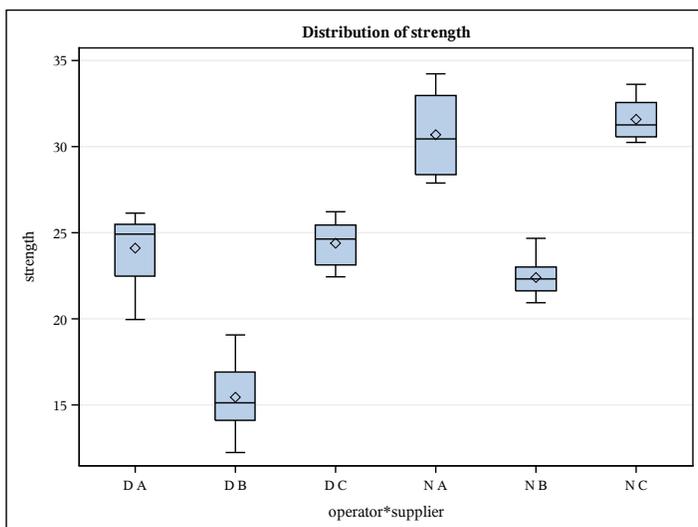




Level of supplier	N	strength	
		Mean	Std Dev
A	24	27.3937500	4.04759597
B	24	18.9170833	3.90416331
C	24	27.9875000	3.87132519



Level of operator	N	strength	
		Mean	Std Dev
D	36	21.3130556	4.59250869
N	36	28.2191667	4.50787969



Level of operator	Level of supplier	N	strength	
			Mean	Std Dev
D	A	12	24.1083333	2.11493749
D	B	12	15.4366667	2.09151678
D	C	12	24.3941667	1.30742953
N	A	12	30.6791667	2.49639907
N	B	12	22.3975000	1.03305483
N	C	12	31.5808333	1.20651608

**The GLM Procedure**  
**Tests of Hypotheses for Mixed Model Analysis of Variance**

**Dependent Variable: strength**

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	operator	1	858.498672	858.498672	26.34	0.0319
	Error	2.1167	68.994864	32.596200		
Error: MS(run(operator)) + MS(operat*batch(suppli)) - MS(run*batc(oper*suppl)) + 12E-16*MS(Error)						
* This test assumes one or more other fixed effects are zero.						

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	supplier	2	1235.831003	617.915501	26.04	0.0339
	Error	2.0756	49.259238	23.732601		
Error: MS(batch(supplier)) + MS(supplie*run(operato)) - MS(run*batc(oper*suppl)) + 89E-17*MS(Error)						
* This test assumes one or more other fixed effects are zero.						

Source	DF	Type III SS	Mean Square	F Value	Pr > F
operator*supplier	2	1.164703	0.582351	0.28	0.8687
Error	0.2263	0.471544	2.083996		
Error: MS(supplie*run(operato)) + MS(operat*batch(suppli)) - MS(run*batc(oper*suppl)) - 89E-17*MS(Error)					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
run(operator)	2	62.152250	31.076125	55.11	0.0012
Error	4	2.255683	0.563921		
Error: MS(supplie*run(operato))					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
batch(supplier)	3	84.889842	28.296614	4.26	0.1325
Error	3	19.944025	6.648008		
Error: MS(operat*batch(suppli))					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
supplie*run(operato)	4	2.255683	0.563921	0.11	0.9745
operat*batch(suppli)	3	19.944025	6.648008	1.30	0.3587
Error	6	30.767600	5.127933		
Error: MS(run*batc(oper*suppl))					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
run*batc(oper*suppl)	6	30.767600	5.127933	19.82	<.0001
Error: MS(Error)	48	12.418933	0.258728		

Source	Type III Expected Mean Square
operator	$\text{Var}(\text{Error}) + 3 \text{Var}(\text{run}*\text{batc}(\text{oper}*\text{suppl})) + 6 \text{Var}(\text{operat}*\text{batch}(\text{suppli})) + 6 \text{Var}(\text{supplie}*\text{run}(\text{operato})) + 18 \text{Var}(\text{run}(\text{operator})) + \text{Q}(\text{operator}, \text{operator}*\text{supplier})$
supplier	$\text{Var}(\text{Error}) + 3 \text{Var}(\text{run}*\text{batc}(\text{oper}*\text{suppl})) + 6 \text{Var}(\text{operat}*\text{batch}(\text{suppli})) + 6 \text{Var}(\text{supplie}*\text{run}(\text{operato})) + 12 \text{Var}(\text{batch}(\text{supplier})) + \text{Q}(\text{supplier}, \text{operator}*\text{supplier})$
operator*supplier	$\text{Var}(\text{Error}) + 3 \text{Var}(\text{run}*\text{batc}(\text{oper}*\text{suppl})) + 6 \text{Var}(\text{operat}*\text{batch}(\text{suppli})) + 6 \text{Var}(\text{supplie}*\text{run}(\text{operato})) + \text{Q}(\text{operator}*\text{supplier})$
run(operator)	$\text{Var}(\text{Error}) + 3 \text{Var}(\text{run}*\text{batc}(\text{oper}*\text{suppl})) + 6 \text{Var}(\text{supplie}*\text{run}(\text{operato})) + 18 \text{Var}(\text{run}(\text{operator}))$
batch(supplier)	$\text{Var}(\text{Error}) + 3 \text{Var}(\text{run}*\text{batc}(\text{oper}*\text{suppl})) + 6 \text{Var}(\text{operat}*\text{batch}(\text{suppli})) + 12 \text{Var}(\text{batch}(\text{supplier}))$
supplie*run(operato)	$\text{Var}(\text{Error}) + 3 \text{Var}(\text{run}*\text{batc}(\text{oper}*\text{suppl})) + 6 \text{Var}(\text{supplie}*\text{run}(\text{operato}))$
operat*batch(suppli)	$\text{Var}(\text{Error}) + 3 \text{Var}(\text{run}*\text{batc}(\text{oper}*\text{suppl})) + 6 \text{Var}(\text{operat}*\text{batch}(\text{suppli}))$
run*batc(oper*suppl)	$\text{Var}(\text{Error}) + 3 \text{Var}(\text{run}*\text{batc}(\text{oper}*\text{suppl}))$

REML Iterations				
Iteration	Objective	Var(run(operator))	Var(batch(supplier))	Var(supplie*run(operato))
0	-29.6208448790	1.5971075231	1.6997371253	0
1	-30.3612722060	1.5688085300	1.8342388043	0
2	-30.3646474818	1.5431435413	1.8042314979	0
3	-30.3646476064	1.5429886668	1.8040504198	0
4	-30.3646476064	1.5429886668	1.8040504198	0

REML Iterations			
Iteration	Var(operat*batch(suppli))	Var(run*batc(oper*suppl))	Var(Error)
0	0.2386969363	1.5292198718	0.2437676874
1	0.5669442373	1.0334825265	0.2571407463
2	0.5576692893	1.0146472254	0.2587180430
3	0.5576133200	1.0145334914	0.2587277801
4	0.5576133200	1.0145334914	0.2587277801

Convergence criteria met.

## VARIANCE COMPONENTS ANALYSIS

### Variance Components Estimation Procedure

REML Estimates	
Variance Component	Estimate
Var(run(operator))	1.54299
Var(batch(supplier))	1.80405
Var(supplie*run(operato))	0
Var(operat*batch(suppli))	0.55761
Var(run*batc(oper*suppl))	1.01453
Var(Error)	0.25873

## SAS Code for Four Factor Example

```
DM 'LOG; CLEAR; OUT; CLEAR;';
```

```
ODS GRAPHICS ON;
ODS PRINTER PDF file='C:\COURSES\ST541\NEST4.PDF';
OPTIONS NODATE NONUMBER;
```

```
DATA comp;
  INPUT operator $ run supplier $ batch strength @@;
```

```
LINES;
```

```
D 1 A 3 21.21   D 1 A 3 21.32   D 1 A 3 19.95   D 1 A 6 23.61
D 1 A 6 24.82   D 1 A 6 24.84   D 1 B 1 15.16   D 1 B 1 16.41
D 1 B 1 15.47   D 1 B 4 12.24   D 1 B 4 13.18   D 1 B 4 12.83
D 1 C 2 24.20   D 1 C 2 23.99   D 1 C 2 23.40   D 1 C 5 22.83
D 1 C 5 22.75   D 1 C 5 22.46   D 2 A 3 25.77   D 2 A 3 26.15
D 2 A 3 26.15   D 2 A 6 25.24   D 2 A 6 25.22   D 2 A 6 25.02
D 2 B 1 19.07   D 2 B 1 17.44   D 2 B 1 18.16   D 2 B 4 15.07
D 2 B 4 15.14   D 2 B 4 15.07   D 2 C 2 25.75   D 2 C 2 25.12
D 2 C 2 25.08   D 2 C 5 26.23   D 2 C 5 25.11   D 2 C 5 25.81
N 3 A 3 27.89   N 3 A 3 27.89   N 3 A 3 28.28   N 3 A 6 33.04
N 3 A 6 31.98   N 3 A 6 31.76   N 3 B 1 23.22   N 3 B 1 22.51
N 3 B 1 24.67   N 3 B 4 21.58   N 3 B 4 21.11   N 3 B 4 20.96
N 3 C 2 32.40   N 3 C 2 31.52   N 3 C 2 31.93   N 3 C 5 30.54
N 3 C 5 30.67   N 3 C 5 30.34   N 4 A 3 29.13   N 4 A 3 28.50
N 4 A 3 28.68   N 4 A 6 32.93   N 4 A 6 33.85   N 4 A 6 34.22
N 4 B 1 23.10   N 4 B 1 22.71   N 4 B 1 22.94   N 4 B 4 22.14
N 4 B 4 21.72   N 4 B 4 22.11   N 4 C 2 30.58   N 4 C 2 30.26
N 4 C 2 31.02   N 4 C 5 33.39   N 4 C 5 33.63   N 4 C 5 32.69
```

```
;
```

```
PROC GLM DATA=comp PLOTS=(ALL);
  CLASS operator supplier run batch;
  MODEL strength=operator|supplier run(operator) batch(supplier)
    supplier*run(operator) operator*batch(supplier)
    run*batch(operator supplier) / SS3;
  MEANS supplier|operator;
  RANDOM run(operator) batch(supplier) supplier*run(operator)
    operator*batch(supplier) run*batch(operator supplier) / TEST;
  TITLE 'QUALITY CONTROL EXAMPLE-- 2 CROSSED AND 2 NESTED EFFECTS';
```

```
PROC VARCOMP DATA=comp METHOD=REML;
  CLASS operator supplier run batch;
  MODEL strength=operator supplier operator*supplier run(operator)
    batch(supplier) supplier*run(operator) operator*batch(supplier)
    run*batch(operator supplier) / FIXED=3;
  TITLE 'VARIANCE COMPONENTS ANALYSIS';
RUN;
```