MA 176 Practice Exam 3 April 7, 2008 NAME_____ Key

~~No Calculators~~

1. Give the equation of a sphere with center (4, 1, -2) and diameter 12.

$(x-4)^2+(y-1)^2+(z+2)^2=36$

2. $e = E\sin 2\pi ft$, find the total differential where π and E are always constant.

$de = 2\pi t\, E\cos(2\pi ft)\,df + 2\pi f E\cos(2\pi ft)\,dt$

3. Find the slope of the tangent line of z = xysiny parallel to the xz and yz planes at $(1, \frac{\pi}{2}, \frac{\pi}{2})$.

$f_x = y\sin y$ slope $= \pi/2$

$f_y = xy\cos y + x\sin y$ slope $= 1$

4. A cylindrical tank has d = 20m and length = 100m. Find the approximate volume of paint (in m^3) required to paint one coat of thickness0.03m over the total exterior.

$r = 10$ m, $l = 100$ m

$V = \pi r^2 h$

$dV = 2\pi rh\,dr + \pi r^2\,dh$

$= 207.3\ m^3$

$dr = .03$, $dh = .06$, $r = 10$, $h = 100$

Evaluate

5. $\int_3^\infty \frac{1}{x^4}\,dx$ $-\frac{1}{3}\left[\lim_{t\to\infty}\frac{1}{t^3} - \frac{1}{3^3}\right] = \frac{1}{81}$

6. $\int_0^2 \frac{1}{x^{3/2}}\,dx$ $-2\left[\frac{1}{\sqrt{2}} - \lim_{t\to 0^+}\frac{1}{\sqrt{t}}\right]$ DV

7. Integrate $\int \frac{x^3}{(x^2+1)^{5/2}}\,dx$

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$\left(\sqrt{\tan^2\theta+1}\right)^5 = \sec^5\theta$

$\sqrt{x^2+1}$, 1, x

$u = \cos\theta$

$\int \frac{\tan^3\theta\sec^2\theta\,d\theta}{\sec^5\theta} = \int \frac{\tan^3\theta}{\sec^3\theta}\,d\theta = \int \sin^3\theta\,d\theta$ $\int \sin(1-\cos^2\theta)$

$= -\left[\cos\theta - \frac{\cos^3\theta}{3}\right] = -\frac{1}{\sqrt{x^2+1}} + \frac{1}{3}\left(\frac{1}{x^2+1}\right)^{+C}$

8. The angle of elevation to the top of a tall yellow pine tree in Montana is 45.0 ±0.01 degrees measured from 100 ±0.11 feet away from the base. Find the height and the maximum possible error.

$\tan\theta = \frac{ht}{dist}$ $ht = dist\tan\theta$

$dht = dist\sec^2\theta\,d\theta + \tan\theta\,d\,dist$

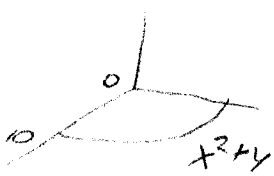
$= 100\sec^2\frac{\pi}{4}\cdot .01\frac{2\pi}{120} + \tan\frac{\pi}{4}(.11$

$100 \pm .14 + .11$

9. Evaluate the double integral: $\int_0^1\int_0^{x^3} e^{y/x}\,dy\,dx$

$x\,e^{x^3/x} - 1$ $\frac{1}{2}e^{x^2} - x\Big|_0^1$

10. Write a double integral to find the volume in the first octant bound by the sphere $x^2 + y^2 + z^2 = 100$ and the planes x = 0 and y = 0. Do NOT evaluate



$y = \sqrt{100-x^2}$

$\int_0^{10}\int_0^{\sqrt{100-x^2}} \sqrt{100-x^2-y^2}\,dy\,dx$