Stat 532 **Name:** ____________________

HW 5

Due 10/2/17

Please prepare your solutions using LaTeX or another word processing software.

1. (5 points) How do Monte Carlo and Markov Chain Monte Carlo (MCMC) procedures differ?

2. (5 points) How are the full conditional distributions used in a Gibbs Sampler?

3. (5 points) How do the marginal and conditional posterior distributions differ?

4. Simulating data is a key step in verifying your algorithms are working correctly. This will be more apparent as we start studying sophisticated hierarchical models.

   (a) (10 points) Simulate 100 observations from a standard normal distribution and plot a histogram of your data.

   (b) (10 points) Select and state prior distributions for θ the mean of the normal distribution and σ^2 the variance such that you can implement a Monte Carlo sampling procedure (hint $\theta \sim N(\mu_0, \sigma/\kappa_0)$

   (c) (20 points) Implement a Monte Carlo procedure to simulate from the joint posterior distribution $p(\theta, \sigma^2|y_1, \ldots, y_{100})$. Create a plot of the joint posterior distribution.

   (d) (20 points) Use your MCMC samples to create a posterior predictive distribution. Compare the data and your posterior predictive distribution using a QQ plot `qqnorm(.)`. Note, you do not need to compute the form of the posterior predictive distribution to sample from it.

5. (25 points) Verify if the prior $p(\theta|\sigma^2) \sim N(\mu_0, \tau_0^2)$ and the sampling model $p(y_1, \ldots, y_n|\theta, \sigma^2) \sim N\left(\theta, \sigma^2\right)$ that the conditional posterior $p(\theta|\sigma^2, y_1, \ldots, y_n)$ is distributed
$N\left(\left(\frac{\mu_0}{\tau_0^2} + \frac{\sum y_i}{\sigma^2}\right)) \left(\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}\right)^{-1}, \left(\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}\right)^{-1}\right).$