Stat 532 **Name:** ____________________

HW 7

Due 10/16/17

Please prepare your solutions using LaTeX or another word processing software.

1. (30 points) For this question we are going to take a deeper look at the impact of our prior distributions and the number of observed data points. Generate fifteen observations from a standard normal distribution and use independent priors on the parameters from the normal distribution: $p(\theta) \sim N(\mu_0, \tau_0^2)$ and $p(\sigma^2) \sim IG(\nu_0/2, \nu_0\sigma_0^2/2)$. This will require running a Gibbs sampler to estimate the posterior distribution.

   First plot the marginal posterior distributions for θ under the flat prior $p(\theta, \sigma^2) \propto 1$. Note that this is proportional to the likelihood function. Next, rerun the Gibbs sampler using at least 5 different values for μ_0, τ_0^2, where some of the values should be close to the truth and some should be very different. The same principles apply for σ^2, but fix $p(\sigma^2) \propto \frac{1}{\sigma^2}$ so that we can see the difference for θ. Make sure to look at a few situations where τ_0^2 is very small and very large. Explain what you have learned from this question and create a figure that contains the prior, likelihood, and posterior on a single graph.

2. (20 points) Assume you are faced with a modeling scenario for a multivariate normal response where you do not have a good sense of the covariance structure. Select and justify a prior distribution for Σ of (Σ^{-1}).

3. (20 points) Verify that for a multivariate normal distribution, the variance and expectation can be extracted from the kernel, $\exp\left[-\frac{1}{2}\left(\tilde{\theta}^t A\tilde{\theta} - \tilde{\theta}^T B + \dots\right)\right]$, where the variance is A^{-1} and the expectation is $A^{-1}B$.

4. (30 points) Simulate data from a multivariate normal distribution, with $p = 3$. Choose a mean vector $\tilde{\mu}$ and covariance matrix Σ. Run a Gibbs sampler and verify that your sampler returns the true values. Comment on the results. Include your code as well.