STAT 436 / 536 - Lecture 13

ARMA Process

- Thus far we have fit either AR(p) processes

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \cdots + \alpha_p x_{t-p} + w_t$$

  or MA(q) processes

$$x_t = w_t + \beta_1 w_{t-1} + \ldots \beta_q w_{t-q}$$

- However, it would be reasonable to want to fit both at the same time. This results an ARMA model.

- ARMA models can be written using the polynomial expression with the characteristic equations:

There are several key points about an ARMA(p,q) process:

a. The process is stationary when the absolute value of the roots of θ all exceed 1.

b. The process is invertible when the absolute value of the roots of ϕ all exceed 1.

c. An AR(p) model is

d. Similarly, the MA(q) model is

e. When fitting to data, an ARMA model will often be more parameter efficient than a single AR or MA model.

f. When θ and ϕ share a common factor, a stationary model can be simplified. For example, with

$$(1 - \frac{1}{2}B)(1 - \frac{1}{3}B)x_t = (1 - \frac{1}{2}B)w_t$$

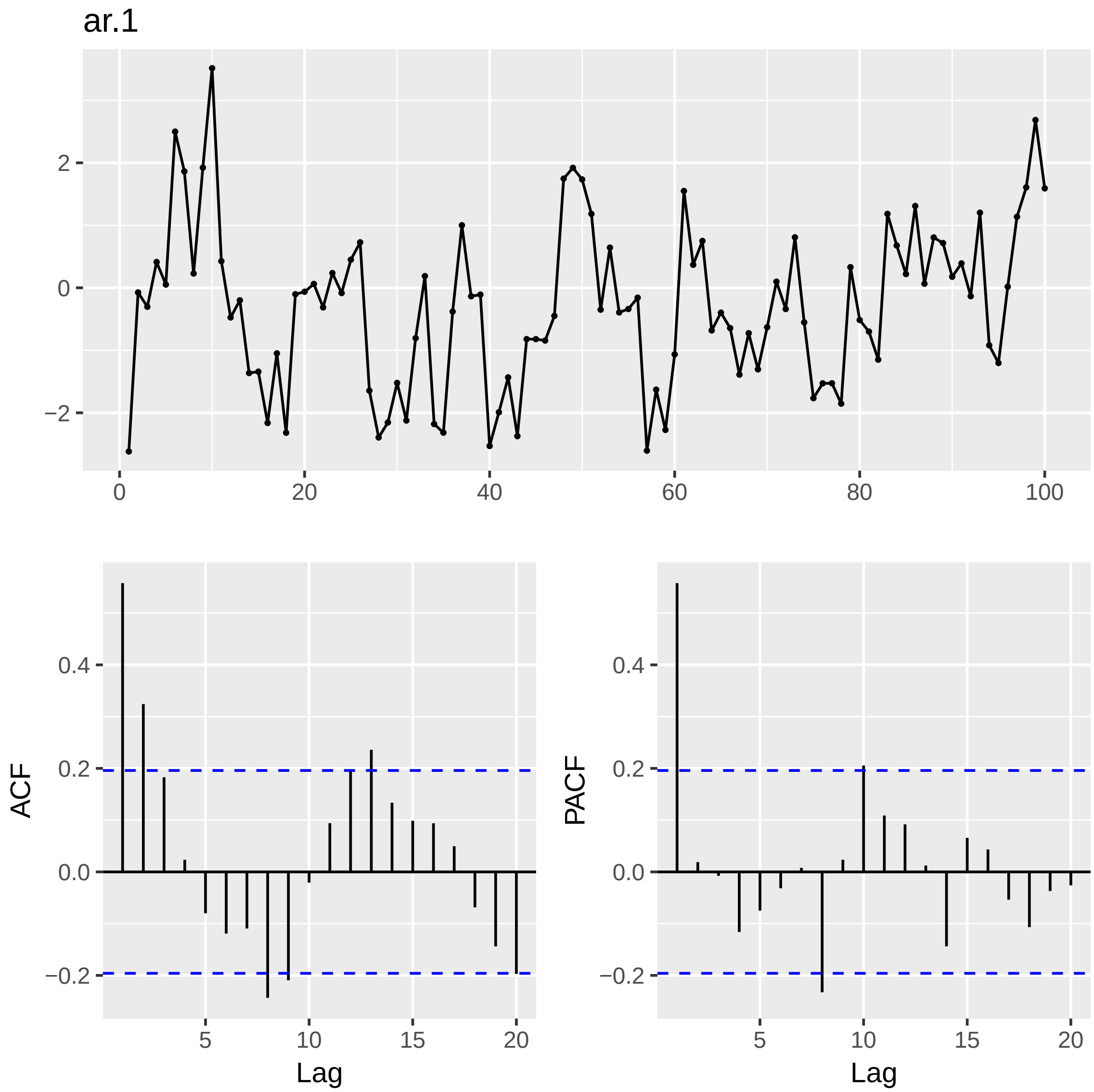
can be simplified to

$$(1 - \frac{1}{3}B)x_t = w_t.$$

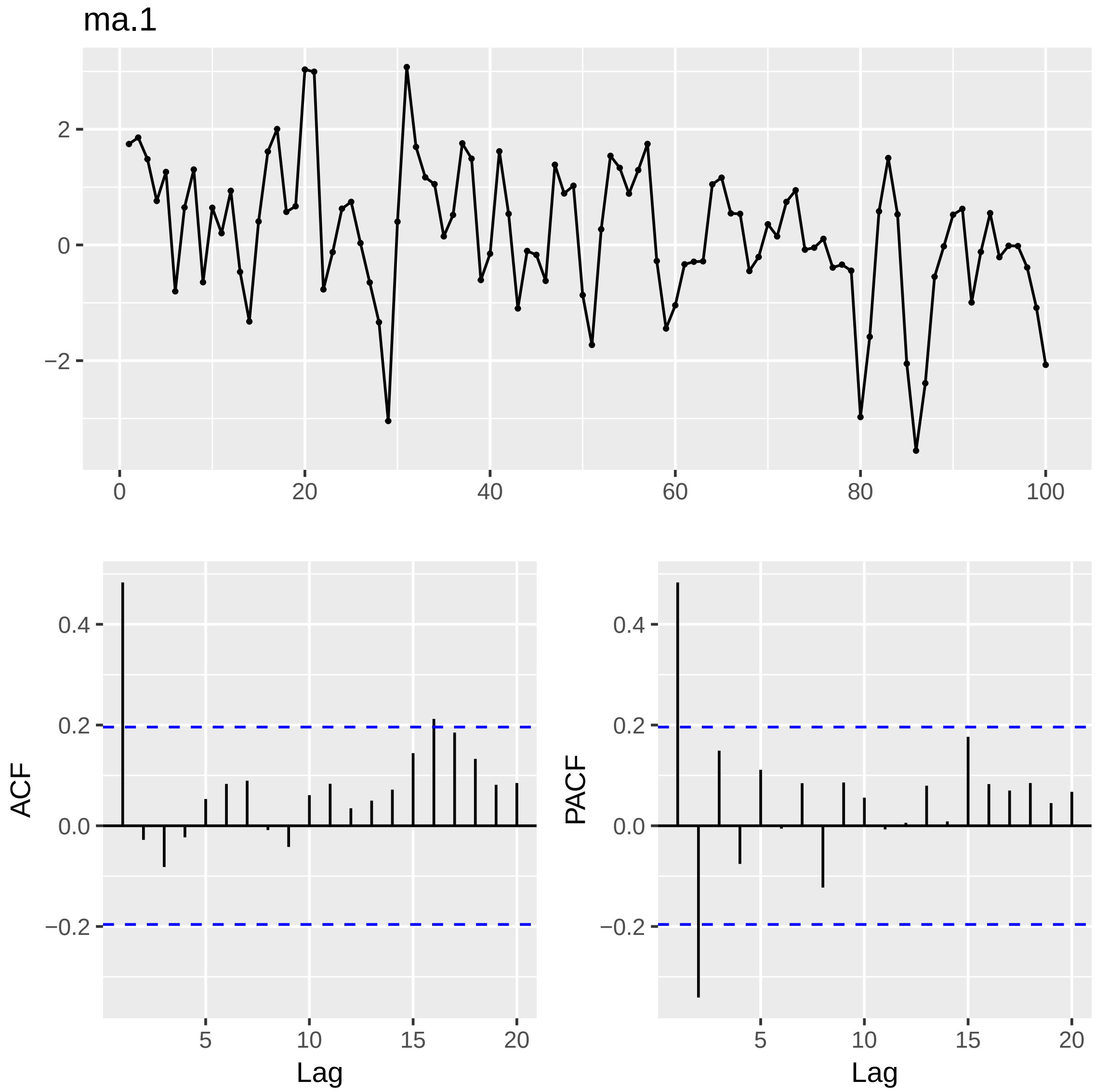
Simulation and Model Fitting

- The r function `arima.sim` allows simulation from ARMA processes.

```r
set.seed(10312018)
#install.packages('fpp')
library(fpp)
ar.1 <- arima.sim(n = 100, model = list(ar = .7))
ggtsdisplay(ar.1)
```




```r
#install.packages('fpp')
library(fpp)
ma.1 <- arima.sim(n = 100, model = list(ma = .7))
ggtsdisplay(ma.1)
```



- Consider fitting the `arma.1` series that is simulated from an AR(1) process using both an AR model and a MA model. How should we choose?

```
arima(ar.1, order = c(1,0,0))
arima(ar.1, order = c(0,0,1))
arima(ma.1, order = c(1,0,0))
arima(ma.1, order = c(0,0,1))
```

- We have seen the predictive framework,

```
auto.arima(ar.1, max.d = 0)
auto.arima(ma.1, max.d = 0)
```