STAT 436 / 536 - Lecture 16

Modeling Non-Stationary Time Series

- Many time series models are non-stationary. Recall a time series is stationary if

- One of our current strategies for non-stationary time series models is to

- When using a regression model, the interest shifts to the residuals. In particular, the residuals should satisfy stationarity.

- We also have seen that differencing a random walk results in a stationary series. A random walk can be written as

$$x_t = x_{t-1} + w_t$$

  and then the differenced series

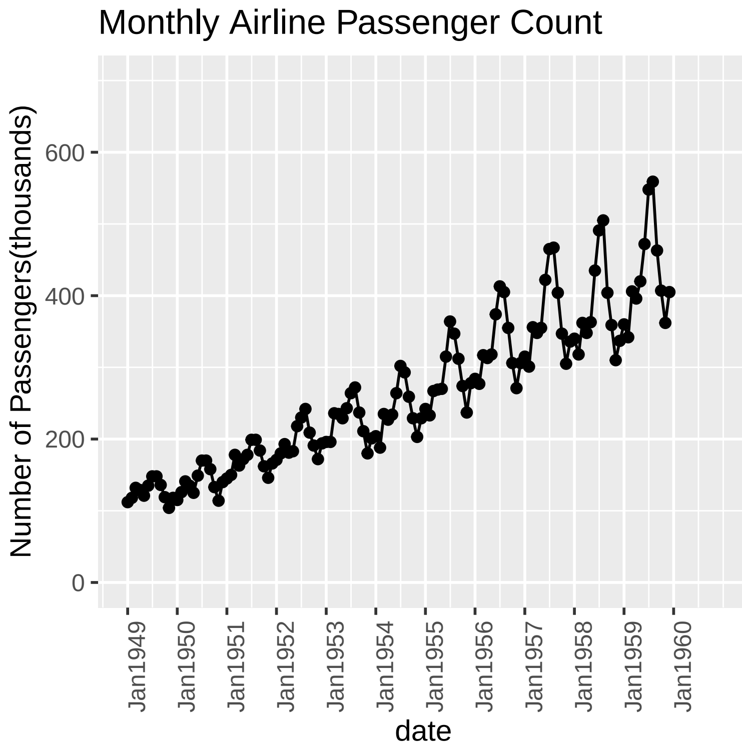
So why is stationarity important?

- Stationarity is a

- When stationarity is not present, differencing (as in the random walk), is used to try to obtain a resulting series that is stationary.

Diagnosing a non-stationary series

- Sometimes a non-stationary series can be diagnosed visually:



- The characteristic equation can be used to determine if a series is stationary, assuming the parameter values are known.

$$
\begin{aligned}
x_t &= x_{t-1} - \frac{1}{4}x_{t-2} + w_t \\
\left(\frac{1}{4}B^2 - B + 1\right) x_t &= w_t
\end{aligned}
$$

```r
polyroot(c(1, -1, .25))
```

```
## [1] 2+0i 2-0i
```

$$
\begin{aligned}
x_t &= x_{t-1} + w_t \\
(B-1)\, x_t &= w_t
\end{aligned}
$$

```r
polyroot(c(-1, 1))
```

```
## [1] 1+0i
```

$$
\begin{aligned}
x_t &= \frac{1}{2}x_{t-1} + \frac{1}{2}x_{t-2} + w_t \\
\left(-\frac{1}{2}B^2 - \frac{1}{2}B + 1\right)x_t &= w_t
\end{aligned}
$$

```
#polyroot()
```

- Another way to diagnose stationarity is to use a unit root test. This is closely related to the idea of a random walk as a unit root corresponds to the solution of the polynomial equation of an AR 1 model.

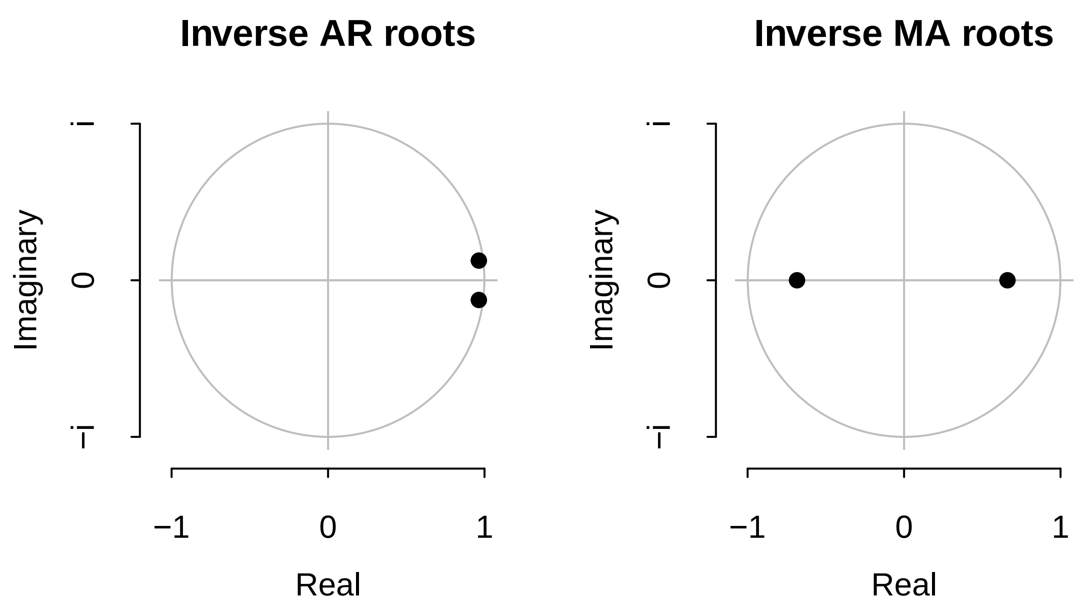
- The `arima.sim()` function will return an error if you try to simulate a non-stationary model.

```
arima.sim(n = 100, list(ar = c(1)))
```

```
## Error in arima.sim(n = 100, list(ar = c(1))): 'ar' part of model is not stationary
```

- The `Arima` and `auto.arima` can be used to assess for stationary with the a fitted model; however, there are restrictions in the model that generally result in non-stationary models being fitted (with the integrated piece).

```
library(forecast)
#autoplot(WWWusage)
plot(auto.arima(WWWusage, stationary = T))
```



- Furthermore, the `auto.arima` package will also select models that "integrate" the data to create a differenced series that will be stationary.

```
auto.arima(WWWusage)
```

```
## Series: WWWusage
## ARIMA(1,1,1)
##
## Coefficients:
##          ar1     ma1
##       0.6504  0.5256
## s.e.  0.0842  0.0896
##
## sigma^2 estimated as 9.995:  log likelihood=-254.15
## AIC=514.3   AICc=514.55   BIC=522.08
```

Integrated Model

- A model is 'integrated' with order d, denoted I(d),

When $d = 1$ this is a random walk.

```
rw <- arima.sim(n=500, list( order = c(0,1,0)))
autoplot(rw)
```


```
auto.arima(rw)
```

```
## Series: rw
## ARIMA(0,1,0) with drift
##
## Coefficients:
##          drift
##        -0.1029
## s.e.    0.0460
##
## sigma^2 estimated as 1.059:  log likelihood=-723.26
## AIC=1450.52   AICc=1450.55   BIC=1458.95
```

- The integrated component can also be combined with ARMA models to form an ARIMA.

$$\theta_p(B)(1-B)^d x_t = \phi_q(B) w_t$$

  is an ARIMA(p, d, q) model.

- For instance,

$$x_t = \alpha x_{t-1} + x_{t-1} - \alpha x_{t-2} + w_t + \beta w_{t-1}$$

- Recall the taxi data set. Run the code below and discuss the results.

```
taxi.rides <- read_csv('http://math.montana.edu/ahoegh/teaching/timeseries/data/taxi.csv')

taxirides.diff <- taxi.rides %>% arrange(year, month, day) %>% slice(-c(1:4)) %>%
  mutate(week.numb = rep(1:234, each = 7)) %>% group_by(week.numb) %>%
  summarize(total.rides = sum(n)) %>% select(total.rides) %>% pull() %>% diff()

auto.arima(taxirides.diff)
```

```
taxirides.summary <- taxi.rides %>% arrange(year, month, day) %>% slice(-c(1:4)) %>%
  mutate(week.numb = rep(1:234, each = 7)) %>% group_by(week.numb) %>%
  summarize(total.rides = sum(n)) %>% select(total.rides) %>% pull()
auto.arima(taxirides.summary)
```

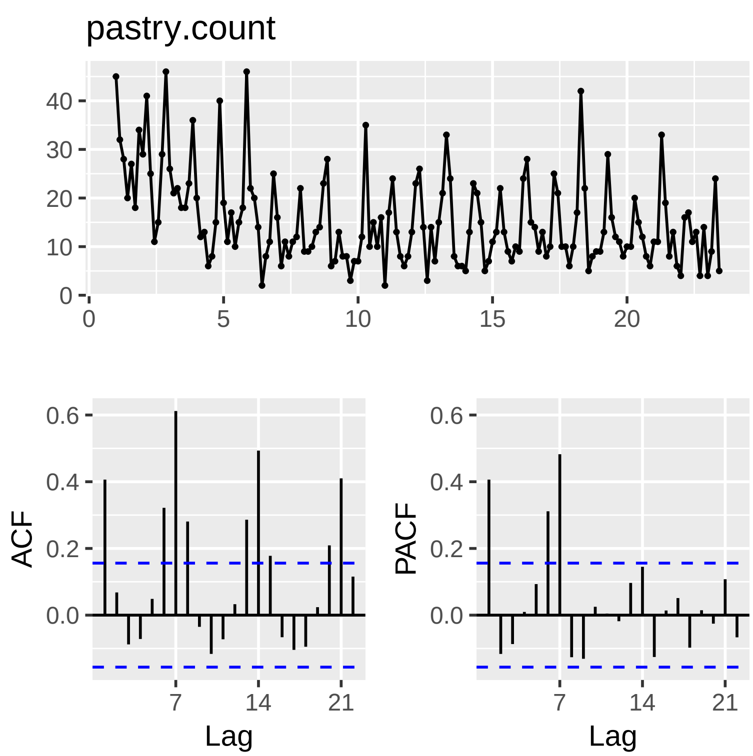
- The `arima` function can also be used to fit a specific order of an ARIMA model.

- As we saw before with ARMA models, AR, ARI, IMA, and ARMA models are all special cases of the ARIMA framework.

Seasonal Arima

- ARIMA models can also have a seasonal component, where the lag corresponds to the seasonal frequency. For example, consider the following model for a time series with weekly seasonal frequency:

```r
library(forecast)
bakery.sales <- read_csv('http://math.montana.edu/ahoegh/teaching/timeseries/data/BreadBasket.csv')
pastry.count <- bakery.sales %>% filter(Item %in% c('Pastry','Scandinavian','Medialuna','Muffin','Scone
ggtsdisplay(pastry.count)
```




```r
auto.arima(pastry.count)
```

```
## Series: pastry.count
## ARIMA(2,0,2)(1,1,1)[7] with drift
##
## Coefficients:
##           ar1      ar2     ma1     ma2     sar1     sma1    drift
##       -0.0402  -0.3802  0.3757  0.5374  -0.0920  -0.4636  -0.1164
## s.e.   0.2783   0.3361  0.2300  0.3482   0.1744   0.1618   0.0504
##
## sigma^2 estimated as 40.24:  log likelihood=-491.05
## AIC=998.11   AICc=999.12   BIC=1022.25
```