1. Circle the appropriate choice of TRUE or FALSE.

   TRUE  FALSE: \( \int \frac{1}{\sqrt{1 - x^2}} \, dx = \arcsin x + C \) (T)

   TRUE  FALSE: \( \int \frac{1}{1 + x^2} \, dx = \ln |1 + x^2| + C \) (F)

   TRUE  FALSE: \( \int \frac{1}{x} \, dx = \ln x + C \) (F)

   TRUE  FALSE: \( \int \sec x \cdot \tan x \, dx = \sec x + C \) (T)

   TRUE  FALSE: \( \int \tan x \, dx = \sec^2 x + C \) (F)

2. Evaluate each integral.

   (a) \( \int e^{\tan x} \sec^2 x \, dx \)  
      ANS (u-sub): \( e^{\tan x} + C \)

   (b) \( \int x \ln x \, dx \)  
      ANS (IBP): \( \frac{x^2}{2} \ln |x| - \frac{x^2}{4} + C \)

   (c) \( \int \frac{x}{2-x^2} \, dx \)  
      ANS (u-sub): \( -\frac{1}{2} \ln |2-x^2| + C \)

   (d) \( \int \frac{1}{x^2 + 4} \, dx \)  
      ANS (recognize arctan form, u-sub): \( \frac{1}{2} \arctan \left( \frac{x}{2} \right) + C \)

   (e) \( \int \arcsin 2x \, dx \)  
      ANS (IBP): \( x \arcsin(2x) + \frac{1}{2} \sqrt{1 - 4x^2} + C \)

   (f) \( \int e^4 \frac{1}{x \sqrt{\ln x}} \, dx \)  
      ANS (u-sub): \( 2 \)

   (g) \( \int \frac{1}{e^x} \, dx \)  
      ANS (IBP): \( 1 - \frac{2}{e} \)

   (h) \( \int_0^{\sqrt{5}} \frac{x}{\sqrt{x^2 + 4}} \, dx \)  
      ANS (u-sub): \( 1 \)

   (i) \( \int_0^{\pi^2} \sin \sqrt{x} \, dx \)  
      ANS (sub: \( w = \sqrt{x} \), IBP): \( 2\pi \)

   (j) \( \int_1^2 \frac{x - 1}{x^2} \, dx \)  
      ANS (manipulate algebraically): \( \ln 2 - \frac{1}{2} \)

   (k) \( \int x \sqrt{1 + x} \, dx \)  
      ANS (u-sub, solve for \( x \)): \( \frac{2}{5} (1 + x)^{5/2} - \frac{2}{3} (1 + x)^{3/2} + C \)
\[(l) \int_0^3 \frac{1 + 2x}{9 + x^2} \, dx \quad \text{ANS (split numerator): } \frac{\pi}{12} + \ln 18 - \ln 9\]

\[(m) \int_{1/8}^{1/4} \frac{1}{\sqrt{1 - 16x^2}} \, dx \quad \text{ANS (recognize arcsin form, u-sub): } \frac{\pi}{12}\]

\[(n) \int x^2 \cos x \, dx \quad \text{ANS (IBP twice): } x^2 \sin x + 2x \cos x - 2 \sin x + C\]

\[(o) \int \frac{x^2}{1 + x^6} \, dx \quad \text{ANS (u-sub with } u = x^3): \frac{1}{3} \arctan(x^3) + C\]

3. Area between curves

(a) Consider the area bounded between the functions \(y = x^2, \ y = x\) for \(0 \leq x \leq 2\). Carefully sketch the graphs and find the area. \textbf{ANS: 1}\n
(b) Consider the area bounded between the functions \(y = \cos x, \ y = \sin 2x\) for \(0 \leq x \leq \frac{\pi}{2}\). Carefully sketch the graphs and find the area. \textbf{ANS: 1/2}\n
4. Volumes

(a) Find the volume of the described solid: The base is the unit circle \(x^2 + y^2 = 1\) and its cross sections are squares perpendicular to the \(y\)-axis. \textbf{ANS: } \frac{16}{3}\n
(b) Find the volume of the described solid: The base is bounded by \(y = \frac{1}{x}\) and the \(x\)-axis for \(x \in [1, 3]\). Cross sections perpendicular to the \(x\)-axis are semicircles. \textbf{ANS: } \frac{\pi}{12}\
5. For the given curves and axes of rotation:

(a) Curves: \( y = x^2 \) and \( y = 2 - x \)
Rotate about \( x \)-axis.

(b) Curves: \( y = e^x \), \( x \)-axis, \( 0 \leq x \leq 2 \)
Rotate about the line \( x = -2 \).

- Washers: \( V = \int_{-2}^{1} \pi \left[ (2 - x)^2 - (x^2)^2 \right] \, dx \)
- Shells: \( \int_{0}^{1} 4\pi y \sqrt{y} \, dy + \int_{1}^{4} 2\pi y (2 - y + \sqrt{y}) \, dy \)
- ANS: Volume: \( \frac{72\pi}{5} \)

- Washers: \( \int_{0}^{1} \pi \left[ 3^2 - 2^2 \right] \, dy + \int_{1}^{e} \pi \left[ 3^2 - (\ln y + 2)^2 \right] \, dy \)
- Shells: \( V = \int_{0}^{1} 2\pi (x + 2)e^x \, dx \)
- ANS: Volume: \( 2\pi(e - 1) \)

6. Work

(a) Calculate the work required to build a cylindrical marble column of height 10 meters and radius 0.5 meters. It may be helpful to know that the density of marble is \( \rho = 2560 \frac{kg}{m^3} \).
ANS: \( \frac{25\pi \rho g}{2} \) J

(b) A tank full of milk (\( \rho = 1030 \frac{kg}{m^3} \)) is in the shape of the graph of \( y = x^4 \) for \( 0 \leq x \leq 2 \) rotated about the \( y \)-axis. Calculate the work required to pump all of the milk out a spout of length 1 m from the top of the tank.
ANS: \( \frac{4736\pi \rho g}{15} \) J

(c) A tank buried in the desert contains a large water supply. The tank is a right circular cylinder with height 4 meters and radius 12 meters (see the diagram). The top of the tank is 2 meters below ground level. If the tank is completely full of water, set up an integral representing the work required to pump all the water out the spigot.
ANS: \( \int_{0}^{4} \rho g \pi (144)(9 - y) \, dy \)