Fall 2017 - M172 Review Session - Sept. 17/19, 2017 - EXAM I

1. Circle the appropriate choice of TRUE or FALSE.

TRUE FALSE: $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$ (T)

TRUE FALSE: $\displaystyle\int \frac{1}{1+x^2}\,dx = \ln|1+x^2| + C$ (F)

TRUE FALSE: $\displaystyle\int \frac{1}{x}\,dx = \ln x + C$ (F)

TRUE FALSE: $\displaystyle\int \sec x \cdot \tan x\,dx = \sec x + C$ (T)

TRUE FALSE: $\displaystyle\int \tan x\,dx = \sec^2 x + C$ (F)

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2. Evaluate each integral.

(a) $\displaystyle\int e^{\tan x}\sec^2 x\,dx$ ANS (u-sub): $e^{\tan x} + C$

(b) $\displaystyle\int x\ln x\,dx$ ANS (IBP): $\dfrac{x^2}{2}\ln|x| - \dfrac{x^2}{4} + C$

(c) $\displaystyle\int \frac{x}{2-x^2}\,dx$ ANS (u-sub): $-\dfrac{1}{2}\ln|2-x^2| + C$

(d) $\displaystyle\int \frac{1}{x^2+4}\,dx$ ANS (recognize arctan form, u-sub): $\dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right) + C$

(e) $\displaystyle\int \arcsin 2x\,dx$ ANS (IBP): $x\arcsin(2x) + \dfrac{1}{2}\sqrt{1-4x^2} + C$

(f) $\displaystyle\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\,dx$ ANS (u-sub): 2

(g) $\displaystyle\int_0^1 \frac{x}{e^x}\,dx$ ANS (IBP): $1 - \dfrac{2}{e}$

(h) $\displaystyle\int_0^{\sqrt{5}} \frac{x}{\sqrt{x^2+4}}\,dx$ ANS (u-sub): 1

(i) $\displaystyle\int_0^{\pi^2} \sin\sqrt{x}\,dx$ ANS (sub: $w = \sqrt{x}$, IBP): 2π

(j) $\displaystyle\int_1^2 \frac{x-1}{x^2}\,dx$ ANS (manipulate algebraically): $\ln 2 - \frac{1}{2}$

(k) $\displaystyle\int x\sqrt{1+x}\,dx$ ANS (u-sub, solve for x): $\dfrac{2}{5}(1+x)^{5/2} - \dfrac{2}{3}(1+x)^{3/2} + C$

(l) $\displaystyle\int_0^3 \frac{1+2x}{9+x^2}\,dx$ ANS (split numerator): $\dfrac{\pi}{12} + \ln 18 - \ln 9$

(m) $\displaystyle\int_{1/8}^{1/4} \frac{1}{\sqrt{1-16x^2}}\,dx$ ANS (recognize arcsin form, u-sub): $\dfrac{\pi}{12}$

(n) $\displaystyle\int x^2 \cos x\,dx$ ANS (IBP twice): $x^2 \sin x + 2x\cos x - 2\sin x + C$

(o) $\displaystyle\int \frac{x^2}{1+x^6}\,dx$ ANS (u-sub with $u = x^3$): $\dfrac{1}{3}\arctan(x^3) + C$

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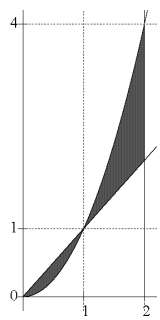
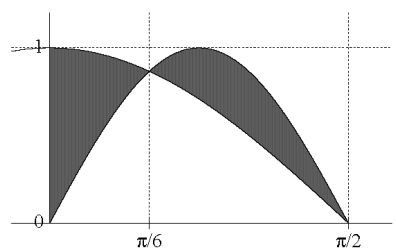
3. Area between curves

(a) Consider the area bounded between the functions $y = x^2$, $y = x$ for $0 \le x \le 2$. Carefully sketch the graphs and find the area. **ANS: 1**



(b) Consider the area bounded between the functions $y = \cos x$, $y = \sin 2x$ for $0 \le x \le \frac{\pi}{2}$. Carefully sketch the graphs and find the area. **ANS: 1/2**



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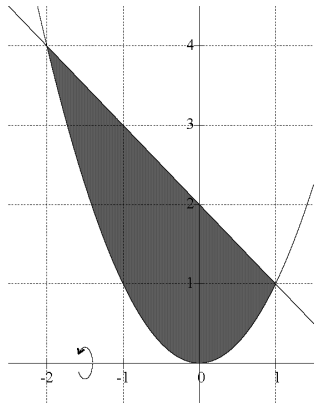
4. Volumes

(a) Find the volume of the described solid: The base is the unit circle $x^2 + y^2 = 1$ and its cross sections are squares perpendicular to the y-axis. ANS: $\dfrac{16}{3}$

(b) Find the volume of the described solid: The base is bounded by $y = \dfrac{1}{x}$ and the x-axis for $x \in [1, 3]$. Cross sections perpendicular to the x-axis are semicircles. ANS: $\dfrac{\pi}{12}$

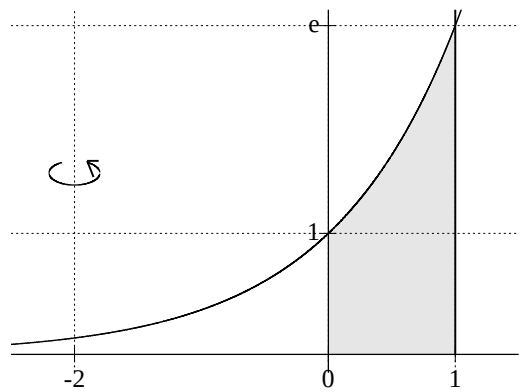
5. For the given curves and axes of rotation:

(a) Curves: $y = x^2$ and $y = 2 - x$
Rotate about x-axis.



- Washers: $V = \int_{-2}^{1} \pi \left[(2-x)^2 - (x^2)^2\right]\, dx$
- Shells: $\int_0^1 4\pi y\sqrt{y}\, dy + \int_1^4 2\pi y(2 - y + \sqrt{y})\, dy$
- ANS: Volume: $\dfrac{72\pi}{5}$

(b) Curves: $y = e^x$, x-axis, $0 \le x \le 2$
Rotate about the line $x = -2$.



- Washers: $\int_0^1 \pi \left[3^2 - 2^2\right]\, dy + \int_1^e \pi \left[3^2 - (\ln y + 2)^2\right]\, dy$
- Shells: $V = \int_0^1 2\pi(x+2)e^x\, dx$
- ANS: Volume: $2\pi(2e - 1)$

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6. Work

(a) Calculate the work required to build a cylindrical marble column of height 10 meters and radius 0.5 meters. It may be helpful to know that the density of marble is $\rho = 2560\frac{kg}{m^3}$.
ANS: $\dfrac{25\pi\rho g}{2} J$

(b) A tank full of milk ($\rho = 1030\frac{\text{kg}}{\text{m}^3}$) is in the shape of the graph of $y = x^4$ for $0 \le x \le 2$ rotated about the y-axis. Calculate the work required to pump all of the milk out a spout of length 1 m from the top of the tank.
ANS: $\dfrac{4736\pi\rho g}{15} J$

(c) A tank buried in the desert contains a large water supply. The tank is a right circular cylinder with height 4 meters and radius 12 meters (see the diagram). The top of the tank is 2 meters below ground level. If the tank is completely full of water, set up an integral representing the work required to pump all the water out the spigot.
ANS: $\int_0^4 \rho g \pi (144)(9 - y)\, dy$