1. Integrate. (Saturday)

(a) \( \int_{0}^{3} t^2 \sqrt{1 + t} \, dt \)  
(b) \( \int \frac{dy}{y^2 + 4} \)  
(c) \( \int_{0}^{\pi/2} x \cos x \, dx \)  
(d) \( \int x \arctan x \, dx \)

2. Integrate. (Monday)

(a) \( \int_{1}^{e^\pi} \frac{\sin (\ln t)}{t} \, dt \)  
(b) \( \int \frac{dy}{\sqrt{2 - y^2}} \)  
(c) \( \int x \ln x \, dx \)  
(d) \( \int_{0}^{3} \frac{1 + 2x}{9 + x^2} \, dx \)

3. Consider the area between the graphs of the functions \( y = \sin x \) and \( y = \cos x \) for \( x \in [0, \pi/2] \). Carefully sketch the graphs and find the area.

4. Find the volume of the solid with base bounded by \( y = |2x| \) and \( y = 2 \) with cross sections perpendicular to the \( y \)-axis that are squares.

5. Consider the region between the graph of \( y = e^x \) and the \( x \)-axis for \( x \in [0, 1] \). Carefully sketch the region. Find the volume of the solid generated by rotating the region about the \( y \)-axis.

6. Consider the region bounded by the graphs of \( y = x^2 \) and \( y = x + 2 \). Carefully sketch the region. Find the volume of the solid generated by rotating the region about the line \( y = -1 \).

7. Calculate the work (in joules) required to pump all of the hydraulic fluid of density \( \rho \) out of a full tank on the planet Tatooine with gravitational constant \( g \). The tank is in the shape of the graph of \( y = x^2 \) for \( x \in [0, 2] \) rotated about the \( y \)-axis. The hydraulic fluid is pumped out a spout of length 1 m from the top of the tank.

8. Integrate. (Saturday)

(a) \( \int \frac{t \, dt}{\sqrt{4 - t^2}} \)  
(b) \( \int_{1/8}^{1/4} \frac{dy}{\sqrt{1 - 16y^2}} \)  
(c) \( \int \frac{\ln x}{x} \, dx \)  
(d) \( \int_{0}^{\pi/4} x \sec^2 x \, dx \)

9. Integrate. (Monday)

(a) \( \int t \sqrt{2 - t} \, dt \)  
(b) \( \int_{0}^{1/3} \frac{dy}{9y^2 + 1} \)  
(c) \( \int_{0}^{1} \frac{1 + 2x}{1 + x} \, dx \)  
(d) \( \int x^2 \sin 2x \, dx \)

10. Consider the area between the graphs of the functions \( y = x^2 \) and \( y = x \) for \( x \in [0, 2] \). Carefully sketch the graphs and find the area.

11. Find the volume of the solid with base bounded by \( y = \frac{1}{2} \) and \( x \)-axis for \( x \in [1, 3] \) with cross sections perpendicular to the \( x \)-axis that are semicircles.

12. Consider the region bounded by the graphs of \( y = x^2 \), \( y = 12 - 4x \), and the \( x \)-axis. Express the volume of the solid generated by rotating the region about the \( x \)-axis as an integral using both the Disk/Washer Method and the Shell Method. Do not evaluate either integral.

13. Consider the region bounded by the graphs of \( x = y^2 \) and \( x = y + 2 \). Carefully sketch the region. Find the volume of the solid generated by rotating the region about the line \( y = 2 \).

14. Calculate the work (in joules) required to pump all of the hydraulic fluid of density \( \rho \) out of a full tank on the planet Tatooine with gravitational constant \( g \). The tank is a horizontal cylinder of radius \( r \) and length \( l \). The hydraulic fluid is pumped out of the top of the tank.
Additional review problems.

1. Integrate.
   
   (a) \( \int \frac{x^2}{x + 1} \, dx \) \hspace{1cm} (b) \( \int_0^1 \frac{x^2}{1 + x^6} \, dx \) \hspace{1cm} (c) \( \int \arcsin x \, dx \) \hspace{1cm} (d) \( \int \frac{\ln \sqrt{x}}{\sqrt{x}} \, dx \)

2. Integrate, slightly harder.
   
   (a) \( \int \frac{x^7 + x^3}{4 + x^8} \, dx \) \hspace{1cm} (b) \( \int e^{2x} \sin 3x \, dx \) \hspace{1cm} (c) \( \int x^2 \arcsin x \, dx \) \hspace{1cm} (d) \( \int \cos^2 x \, dx \) [IBP]

3. Consider the area bounded by the graphs of the functions \( y = x^2 - 2x - 3 \) and \( y = 21 - x^2 \). Carefully sketch the graphs and find the area.

4. A solid has base bounded by the graphs of \( y = \frac{2}{1+x} \), \( y = x \), and \( x = 0 \) with cross sections perpendicular to the \( x \)-axis that are square.
   
   (a) Express the volume of the solid as an integral.
   
   (b) Evaluate the integral. [WARNING! This requires some tricky algebra.]

5. The region bounded by the graphs of \( y = x^2 \) and \( y = 2 - x \), the shaded region in Figure 1 below, is revolved around the line \( x = 2 \). Express the volume of the resulting solid as an integral, or integrals, using the Disk Method and the Shell Method. Evaluate the easier of the two.

6. The region in the first quadrant bounded by the graphs of \( y = 5 - x^2 \) and \( y = 6/x - 2 \), the shaded region in Figure 2 below, is revolved around the line \( x = -1 \). Express the volume of the resulting solid as an integral using the Disk Method and the Shell Method. Evaluate the easier of the two.

7. Show the volume of a sphere of radius \( R \) is \( V = \frac{4}{3} \pi R^3 \).

8. Calculate the work (in joules) to build a pyramid with square base of side length \( s \) and height \( h \) out of a material of density \( \rho \) with a gravitational constant \( g \).
9. After a long climb to the top of a beanstalk of height 800 m Jack sees a giant drinking a murky red cocktail. The interior of the glass is in the shape of a frustum with lower radius 1 m and upper radius 2 m. The height of the interior of the glass is 4 m. The Blood of an Englishman\(^1\) cocktail she is drinking has a density of \(\rho\), and the giant is sipping her cocktail using a straw that extends 2 m past the top of the glass. When Jack arrived the giant had already consumed the ‘top half’ (as measured by height, not volume). How much work does the giant do by slurping the ‘bottom half’ up through the straw? You may assume the acceleration due to gravity at the top of the beanstalk is given by \(g_o\).

Express your solution as a definite integral; do not evaluate the integral. Please start by clearly identifying the coordinate system that you will be using.

Extra Challenge: Suppose the giant took a short nap before finishing her Blood of an Englishman cocktail so that it is now very poorly mixed and has a density given by \(\rho(y) = \kappa(12 - y)\) where \(y\) measures the distance from the bottom of the interior of the glass. How much work is now required to slurp the ‘bottom half’ up through the straw? Again, express your solution as a definite integral, but do not evaluate the integral.

\(^1\)No Englishmen were harmed in the writing of this question. No blood of any kind was used for her cocktail; the giant is a very pleasant vegan.