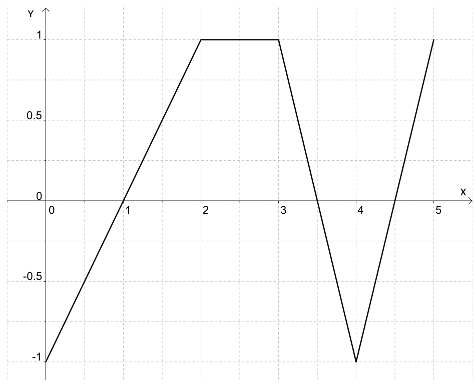
Problem 1. Consider the function $f(x) = x^4 - 8x^2 + 20x + 10$. Use the Intermediate Value Theorem to show that the graph of f has a horizontal tangent line between $x = -3$ and $x = -2$.

Problem 2. Consider the graph below of the function $f(x)$ on the interval $[0, 5]$.



1. For which x values would the derivative $f'(x)$ not be defined?
2. Sketch the graph of the derivative function f'.

Problem 3. The Monod Growth function is given by

$$G(S) = \mu \frac{S}{K + S}$$

where μ and K are constant.

A) Find $G'(S)$.

B) Find the equation of the line tangent to the curve $y = G(S)$ at $S = 0$.

C) Find $\lim\limits_{S \to \infty} G'(S)$.

Problem 4.

A) Find the equation of the line tangent to the curve $y = \frac{x}{x+1}$ at $x = 0$.

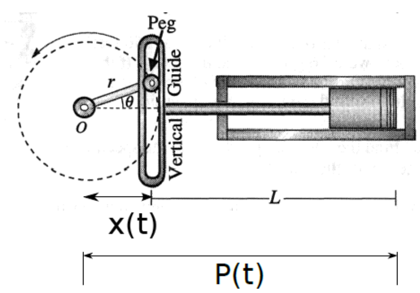
B) Find an equation of the tangent line to $y = 2e^x$ at $x = 0$ and another tangent line at $x = \ln(2)$.

Problem 5. Let $f(x) = (3x - 1)e^x$. For which x is the slope of the tangent line to f positive? Negative? Zero?

Problem 6. Suppose that $f(2) = 3,\ g(2) = 2,\ f'(2) = -2,,\ g'(2) = 4$ and $f'(16) = 0$. For the following functions, find $h'(2)$.

1. $h(x) = 5f(x) + 2g(x)$
2. $h(x) = f(x)g(x)$
3. $h(x) = \dfrac{g(x)}{1 + f(x)}$
4. $h(x) = \sqrt{[f(x)]^2 + 7}$.
5. $h(x) = f(x^3 \cdot g(x))$.

Problem 7. A peg on the end of a rotating crank slides freely in the vertical guide shown in the diagram. The guide is rigidly connected to a piston which moves horizontally. The crank rotates at a constant angular speed ω in a circle of radius r.



It is easy to show that the position of the piston is given by

$$P(t) = L + r\cos(\omega t),$$

since $x(t) = r\cos(\omega t)$. Find the velocity and acceleration of the piston: $\frac{d}{dt}P(t)$ and $\frac{d^2}{dt^2}P(t)$.

Problem 8. If a fish is swimming at speed v against a current with speed c ($c < v$) the amount of energy required to swim a fixed distance L is given by

$$E(v) = \frac{aLv^2}{v - c}$$

Find the rate change of Energy with respect to speed v.

Problem 9. Let

$$g(t) = \begin{cases} t^2 + b & \text{if } t \leq 0 \\ at + 4 & \text{if } t > 0 \end{cases}.$$

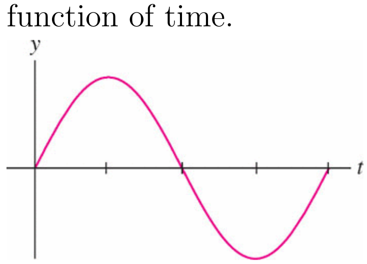
Find a and b so that g is differentiable at $t = 0$.

Problem 10. By Faraday's Law, if a conducting wire of length l meters moves at a velocity v m/s perpendicular to a magnetic field of strength B (in teslas), a voltage of size $E = -Blv$ is induced in the wire.

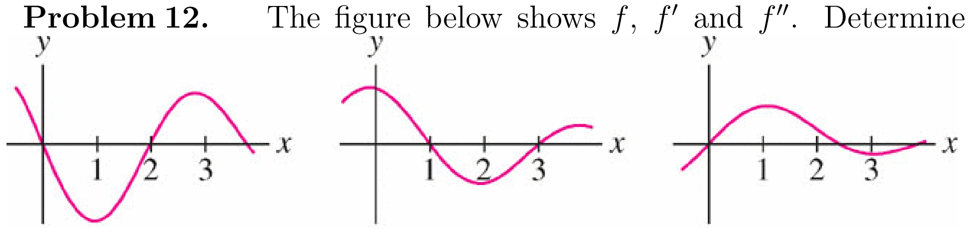
A) Calculate $\frac{dE}{dv}$

B) Suppose that the wire is oscillating in such a way the $v = 10\sin(\omega t)$ where ω is a positive constant. Find the rate of change of E with respect to time t.

Problem 11. The figure shows the height y of a mass oscillating at the end of a spring, through one cycle of the oscillation. Sketch the graph of the velocity as a function of time.



Problem 12. The figure below shows f, f' and f''. Determine which is which.



(A) (B) (C)

Problem 13. Find a formula for the n th derivative of $f(x) = e^{-2x}$.

Problem 14. Find the equation of the line tangent to the curve $y = A\tan(\omega x)$ at $x = \frac{\pi}{3\omega}$. Here A and ω are constants.

Problem 15. Find the derivative of y with respect to x:

1. $e^{(xy)} = \cos(y^4)$.
2. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = \pi^{\frac{2}{3}}$.

Problem 16. Find an equation of the tangent line to the curve

$$x^2 + \sin(y) = xy^2 + 1$$

at the point $(1, 0)$.

Problem 17. The Lambert function, $W(x)$, is implicitly defined by the following equation:

$$W(x)e^{W(x)} = x.$$

Use implicit differentiation to find a formula for $\frac{d}{dx}W(x)$.

Problem 18. Find y' using logarithmic differentiation of $y = x^{\cos(x)}$.

Problem 19. Compute the first derivative of each of the following functions:

1. $f(x) = \cos(4\pi x^3) + \sin(3x + 2)$
2. $b(t) = t^4\cos(3t^2)$
3. $y(\theta) = e^{\sec(2\theta)}$
4. $k(s) = \ln(7s^2 + \sin(s) + 1)$
5. $u(x) = (\sin^{-1}(2x))^2$
6. $h(x) = \dfrac{8x^2 - 7x + 3}{\cos(2x)}$

Problem 20. Compute the first derivative of each of the following functions:

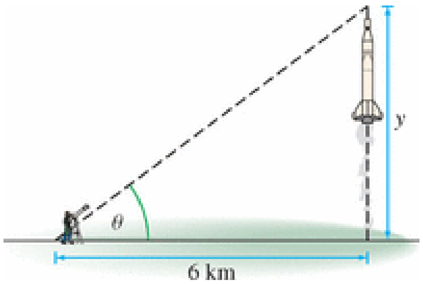
7. $m(x) = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}}$
8. $q(x) = \dfrac{e^x}{1 + x^2}$
9. $n(x) = \cosh(\tan(x))$
10. $w(x) = \arctan(x^2 + 1)$
11. $w(x) = x^{(x^2+1)}$
12. $x(t) = Ae^{-kt}\cos(\omega t + b)$

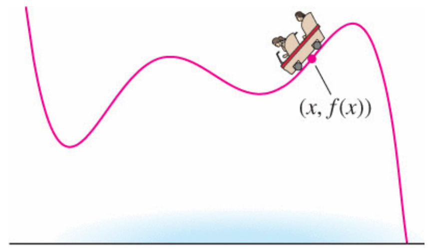
Problem 21. Compute the first derivative of each of the following functions:

13. $P(t) = 2^t$
14. $E(y) = y\ln(1 + e^{y^2})$
15. $l(w) = \sinh(\cos(w))$
16. $k(x) = \ln(7x^2 + 2x + 1)$
17. $q(x) = \dfrac{\ln(x)}{1 + x}$
18. $g(t) = \cosh(at^2 + b)$
19. $g(t) = \sinh(4t + 2)$

Problem 22. As Claudia walks away from a 264-cm lamppost, the tip of her shadow moves twice as fast as she does. What is Claudias height?

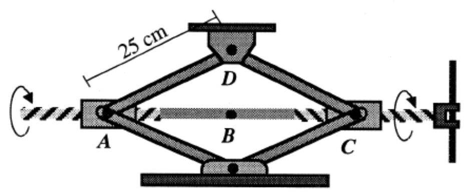
Problem 23. A spy uses a telescope to track a rocket launched vertically from a launching pad 6 km away, as in the Figure below. At a certain moment, he angle between the telescope and the ground is equal to $\dfrac{\pi}{4}$ and is changing at a rate of 1 rad/min. What is the rocket's velocity at that moment?





Problem 24. A roller coaster has the shape of the graph in the Figure above. Show that when the roller coaster passes the point $(x, f(x))$, the vertical velocity of the roller coaster is equal to $f'(x)$ times its horizontal velocity, $x'(t)$.

Problem 25. The height of the automobile jack shown is controlled by rotating a screw ABC which is single-threaded at each end, the pitch is 2.5 mm. If the screw ABC is rotated at a rate of 10 rpm clockwise at C, then one can show that the length of AB DECREASES at the constant rate of 2.5 cm/min, i.e., $dx/dt = -2.5$. (B is the midpoint of AC)

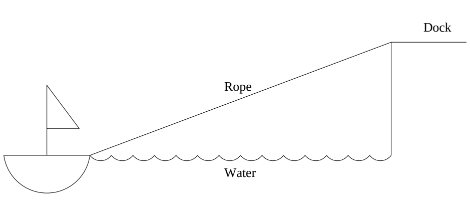


(a) When the length AB is 20 cm, at what rate is the length BD changing? [i.e when $x = 20$ what is dy/dt]? (Note: $25^2 = 20^2 + 15^2$).

(b) When the length AB is 20 cm, what is acceleration of the length BD assuming that x is decreasing at a constant rate, [i.e, find d^2y/dt^2].

(c) When the length AB is 20 cm, at what rate would a car sitting on the jack be moving upward? (Hint The height of the car is a constant plus $2y$)

Problem 26. A boat is drawn close to a dock by pulling in a rope as shown. How is the rate at which the rope is pulled in related to the rate at which the boat approaches the dock?



a. One is a constant multiple of the other.

b. They are equal.

c. It depends on how close the boat is to the dock.