Practice Questions for Exam 4 - Section 4.1-4.5 and 4.7-4.8

Problem 1. Multiple Choice. Peeling an orange changes its volume V, what does ΔV represent?

a. the volume of the rind
b. the surface area of the orange
c. the volume of the "edible part"

Problem 2. Multiple choice: Suppose that $f''(x) < 0$ for x near a point a. Then the linearization of f at a is

1. an over approximation
2. an under approximation
3. unknown without more information.

Problem 3.

a. Find the linear approximation to $\sqrt{a+x}$ for x near zero. Here a is a positive constant.
b. Use your approximation to estimate $\sqrt{4+0.1}$

Problem 4. You want to calculate the depth of a fissure on Mars from the equation $s(t) = 4t^2$ by timing how long it takes a heavy stone you drop to hit the ice below. Here $s(t)$ is the distance in meters an object falls in t seconds under the influence of gravity on Mars. Suppose you measure the time to be 5 seconds. Use differentials to estimate the error in depth if there is a 0.1 second inaccuracy in measuring the time.

Problem 5. Find the linear approximation for $h \approx 0$ for

$$w(h) = \frac{W}{(1+h/R)^2},$$

which is the weight of a body at altitude h above the earth's surface, where W is the weight of the object of the body at the surface of the earth and R is the radius of the earth.

Problem 6. The quadratic approximation to a function $f(x)$ at $x = a$ is given by

$$Q(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2.$$

Find the quadratic approximation to $f(x) = \ln(1+x)$ at $x = 0$ and use it to approximate $\ln(1.1)$

Problem 7. Recall that Newton's method is used to find approximate solutions to $f(x) = 0$ by using the iteration: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. Show how to setup Newton's method to find the cube root of b, i.e. show how would to setup Newton's method to compute a solution of $x^3 = b$.

Problem 8. Explain why Newton's method eventually fails when finding zeros of $f(x) = x^3 - 3x + 7$ with a starting value $x_1 = 2$.

Problem 9. The amount of energy used by a trout to swim at speed v against a current with speed U a distance L is given by

$$E(v) = L\frac{v^3}{v - U}$$

as long as $v > U$. Find the speed that minimizes the energy E. Be sure to show that you do have a min.

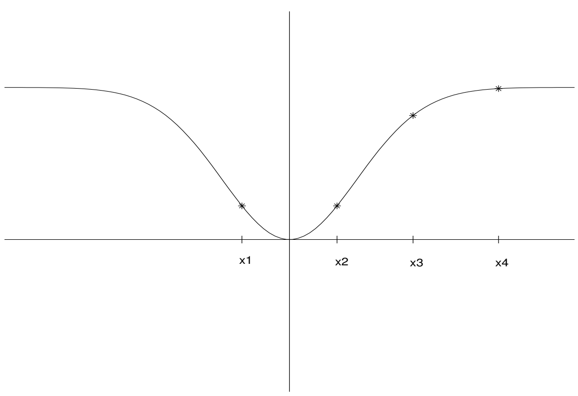
Problem 10. The Reaction $R(x)$ of a patient to a drug dose of size x depends on the type of drug. For a certain drug, it was determined that a good description of the relationship is:

$$R(x) = Ax^2(B - x), \qquad 0 \le x \le B$$

where A and B are positive constants. The Sensitivity of the patients body to the drug is defined to be $R'(x)$.

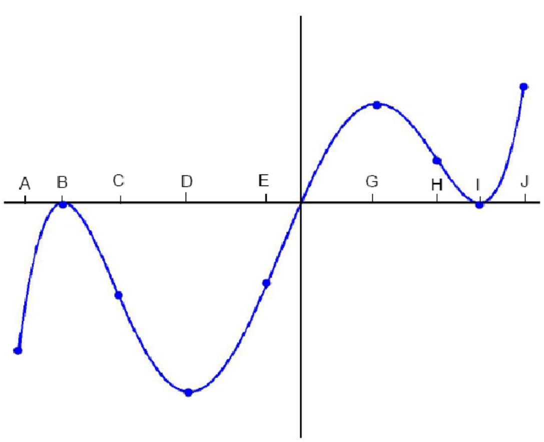
a. For what value of x is the reaction a maximum, and what is that maximum reaction value?
b. For what value of x is the sensitivity a maximum? What is that maximum sensitivity?

Problem 11. We will use each of the x_n below as the starting point for Newton's method. For which of them do you expect Newton's method to work and lead to the root of the function?



1. x_1 and x_2 only.
2. x_2 only.
3. x_1, x_2 and x_3 only.
4. All four.

Problem 12. The following graph is the graph of $f'(x)$. Where do the points of inflection of $f(x)$ occur and on which intervals is f concave up? Does f have a local max or min at $x = 0$?



Problem 13. Multiple Choice. The second derivative, $f''(x)$, of a function $f(x)$ is negative everywhere. We know that $f(0) = 0$ and $f'(0) = 0$. What must be true about $f(1)$?

a. $f(1)$ is negative
b. $f(1)$ is positive
c. $f(1)$ is zero
d. Not enough information to conclude anything about $f(1)$.

Problem 14. Find the interval(s) on which f is concave up or down, the points of inflection, the critical points, and local minima and maxima for $f(x) = \frac{x}{\ln(x)}$.

Problem 15. Find all critical points of the following functions (if they exist)

a. $f(x) = \dfrac{x^2}{2} - 3\sqrt[3]{x}$
b. $f(x) = (x^2 - 4)^{\frac{1}{3}}$
c. $f(x) = xe^{-x}$
d. $f(x) = \dfrac{x^2}{1+x}$

Problem 16. Sketch the graph of a function for which $f'(x) < 0$ for $x < -2$, $f'(x) > 0$ for $x > -2$, $f''(x) > 0$ for $x < 0$, and $f''(x) < 0$ for $x > 0$.

Problem 17. Given that $f(x) = \frac{x^2}{x^2+1}$, $f'(x) = \frac{2x}{(x^2+1)^2}$, and $f''(x) = \frac{2-6x^2}{(x^2+1)^3}$. Make a sketch of the graph the function f. Be sure to include local extrema and points of inflection.

Problem 18. Show that $\sqrt{1+x} < 1 + \frac{1}{2}x$ for $x > 0$.

Problem 19. The fastest drag racers can reach a speed of 300 mi/hr over a quarter-mile strip in 4.45 seconds (from a standing start), Complete the following sentence about such a drag racer: At some point during the race, the maximum acceleration of the drag racer is at least ______ mi/hr/sec.

Problem 20. In general, a lapse rate is the negative of the rate of temperature change with altitude change, thus:

$$\gamma = -\frac{dT}{dz}$$

where γ is the lapse rate given in units of temperature divided by units of altitude, $T =$ temperature, and $z =$ altitude. If the lapse rate exceeds $7°C$ / km in a layer of the atmosphere, it is favorable for thunderstorms and tornado formation . If concurrent measurements indicate that at an elevation of 6 km, the temp is $-20°C$ and at 3 km the temp is $10°C$, can you conclude that the lapse rate exceeds the threshold value of $7°C$ at some intermediate elevation? Explain.

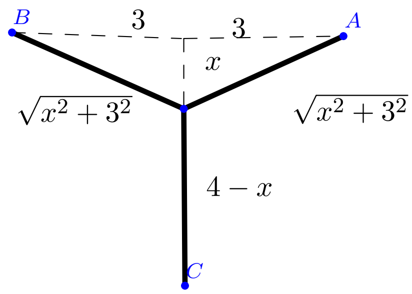
Problem 21. Evaluate the following limits.

a. $\displaystyle\lim_{x\to\pi} \frac{x-\pi}{\sin(x-\pi)}$
b. $\displaystyle\lim_{x\to\infty} x^2 \sin\left(\frac{1}{2x^2}\right)$
c. $\displaystyle\lim_{x\to 0^+} \frac{x}{\ln(x)}$
d. $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x+1}}{\sqrt{2x+1}}$
e. $\displaystyle\lim_{x\to\pi/2} \frac{1-\sin(x)}{\cos(x)}$
f. $\displaystyle\lim_{x\to\infty} \frac{x\ln(x)}{x^a}$ with $a > 1$.
g. $\displaystyle\lim_{h\to 0} \frac{e^h - 1}{h}$

Problem 22. Compute the following limit using L'Hopital's rule:

$$\lim_{\omega\to 1} \frac{1}{\omega^2 - 1}[\sin(\omega t) - \omega\sin(t)]$$

Problem 23. The bottom of the legs of a three-legged table are the vertices of an isosceles triangle with sides 5,5,6. The legs are to be braced at the bottom by three wires in the shape of a **Y**. What is the minimum length of wire needed? Show it is a minimum.

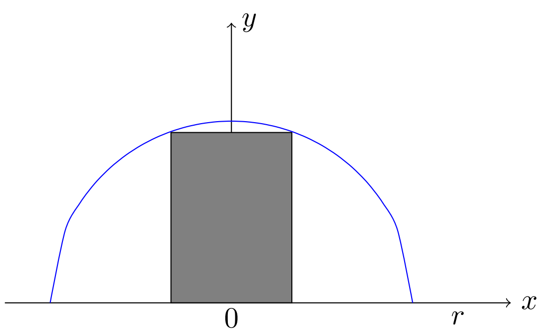


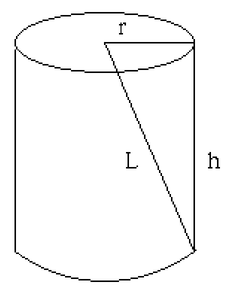
Problem 24. The power transmitted by a pulley belt system is given by the following equation

$$P(v) = (Fv - \rho A v^2)(1 - e^{-\mu\theta})$$

Where P is the power, v is the velocity of the belt, ρ is the density of the belt, A is the cross sectional area of the belt, F is the maximum force in the belt, μ is the coefficient of friction, and θ is the contact angle. Assuming the P is a function of v and that all the other parameters are constant and positive, determine the velocity that makes the power a maximum, and find a formula for this maximum power.

Problem 25. An architect plans to build a rectangular window under a arch that is a semi-circle. Find the maximum area of a rectangle inscribed in semi-circle of radius r.





Problem 26. In 1613, Kepler bought a barrel of wine for his wedding but questioned the method the wine merchant used to measure the volume of the barrel and thus determine the price. The price for a barrel of wine was determined solely by the length L of a dipstick that was inserted diagonally through a centered hole in the top of barrel to the edge of the base of barrel (see figure above).

In consequence, afterwards Kepler set out to find the proportions that optimize the volume of such a barrel. Suppose that the wine barrel is a cylinder as shown. Determine the ratio of the radius r to the height h of the barrel that maximize the Volume of the barrel subject to the wetted length L of the dipstick is fixed.