Math 171 Written Homework 5 Name ____________________

Sections 4.1 and 4.8

October 27, 2017

Credit given for work shown.

Suppose that f is a function of x. If $x = x_0 + \Delta x$, then we can define $\Delta f = f(x_0 + \Delta x) - f(x_0)$. The calculation $\Delta f / \Delta x$ would measure how much a change in x would affect the value of f. This is a simple example of what is called sensitivity analysis.

Since $f'(x_0) = \lim_{\Delta x \to 0} \dfrac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$, an approximation for the sensitivity at a specific x_0 is given by

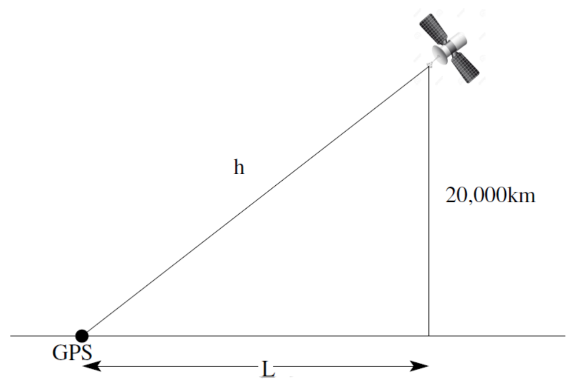
$$\frac{\Delta f}{\Delta x} \approx f'(x_0)$$

and this is what is used in many engineering applications.

1. [4 pts] **Sensitivity of Measurements:**

The planet Quirk is flat. GPS satellites hover over Quirk at an altitude of 20,000 km. See how accurately you can estimate the distance L from the point directly below the satellite to a point on the planet surface knowing the distance h from the satellite to the point on the surface in two cases. (The letter h is for hypotenuse.) With GPS, radio signals give us h up to a certain measurement error (See Figure). The question is how accurately can we measure L. To decide, we find $\frac{\Delta L}{\Delta h}$.
We see that

$$L(h) = \sqrt{h^2 - (20000)^2}$$



a. Use your phone, a calculator or a spread sheet to compute $\Delta L / \Delta h$ for $h = h_0 \pm \Delta h = 25,000 \pm \Delta h$, and $\Delta h = 1, 10^{-1}, 10^{-2}$. (The first case is filled in for you, double check it.)

Δh	-1	1	-0.1	0.1	-0.01	0.01
ΔL	-1.667					
$\Delta L/\Delta h$	1.667					

Once you fill out the table below, write an estimate of L in the form

$$|L(h_0 + \Delta h) - L(h_0)| = |\Delta L| \leq C|\Delta h|$$

choosing the simplest integer C that works for all six cases. $C =$ ____________

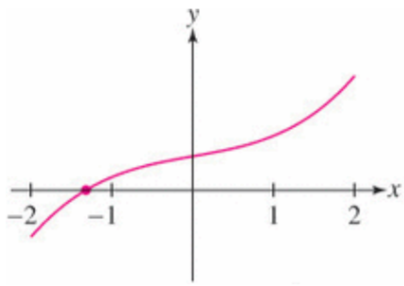
b. Do the same for $h = 20,001 \pm \Delta h$, and $\Delta h = 1, 10^{-1}, 10^{-2}$.

Δh	-1	1	-0.1	0.1	-0.01	0.01
ΔL	-200.002					
$\Delta L/\Delta h$	200.002					

$C =$ ____________

c. Compute $L'(h_0)$ and compare your values of C that you got in part (a) and (b) above to $L'(h_0)$ with $h_0 = 25,000$ for (a) and $h_0 = 20,001$ for (b).

2. [2 pts]



Pick any positive number for x_0. Let x_1, x_2 be the estimates to a root obtained by applying Newton's Method with your choice for x_0 to the function graphed below. Label x_1 and x_2 on the coordinate system, and draw the tangent lines used to obtain them.

3. [4 pts] Recall that the linear approximation $L(x)$ of f is given by the following:

$$L(x) = f(a) + f'(a)(x - a).$$

A quadratic approximation is an extension of linear approximation. It is created by adding one more term related to the second derivative to the aboves. The formula for the quadratic approximation $Q(x)$ of a function $f(x)$ near a is given by

$$Q(x) = \underbrace{f(a) + f'(a)(x - a)}_{\text{Linear Part}} + \underbrace{\frac{f''(a)}{2}(x - a)^2}_{\text{Quadratic Part}}.$$

This approximation is more complicated than the linear approximation and is only used when higher accuracy is needed.

In particular, consider the function $f(x) = \sqrt{x + 1}$.

(a) Find $\sqrt{1.1} =$ ______________ with a calculator.

(b) Determine the linearization of $f(x) = \sqrt{1 + x}$ near $x = 0$

(c) Using the linear approximation from (b), approximate $\sqrt{1.1}$.

(d) Find the quadratic approximation to $f(x) = \sqrt{1 + x}$ at $x = 0$.

(e) Using the quadratic approximation, approximate $\sqrt{1.1}$.

(f) Circle one: Which is closer to the actual value? Linear or Quadratic

(g) Find the formula for the quadratic approximation to $f(x) = (1 + x)^\alpha$ at $x = 0$.