

**Math 171 Group Worksheet 1**  
 on Chapter 2 ideas  
 Due Monday, September 17, 2018

One submission per group. Grade will be based on mathematical processes, precision, proper notation and participation. Make sure to write legible and explain your steps. If needed, staple multiple pages together before handing in work. You may also submit electronically through D2L.

**Problem 1.** Design a roller coaster  $r(x)$  with the following conditions:

- the roller coaster starts on the ground  $r(0) = 0$ .
- the roller coaster cannot exceed the height of 80 meters:  $r(x) \leq 80$  for all  $x \in [0, 100]$
- the roller coaster does not go underground:  $r(x) \geq 0$  for all  $x \in [0, 100]$ .
- the ride is connected. No breaks, gaps or holes!
- at least three equations for different types of functions (logarithmic, exponential, trigonometry, rational, polynomial, etc)

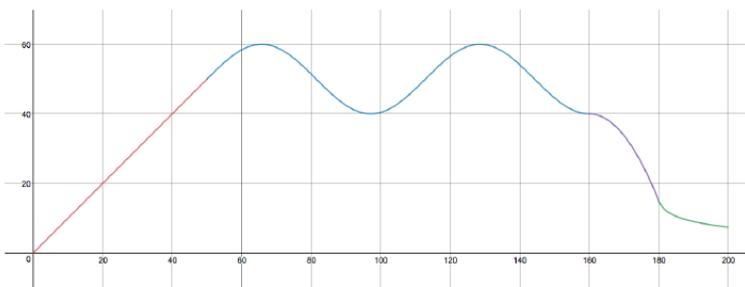
Be sure to include the following in your problem:

- (1) your equation for the graph of the roller coaster listed as a piecewise function
- (2) a graph of the roller coaster is produced using either technology or well-detailed hand drawn
- (3) verification of all conditions
- (4) a report that is well-organized and easy to follow and computations are presented neatly by hand or typed

Grading on Problem 1 will be on the following criteria:

Item	Criteria	Points
1.	The piecewise function $r(x)$ is clearly listed and contains at least three different types of functions (logarithmic, exponential, trigonometric, rational, polynomial, etc.)	4
2.	A graph of the roller coaster is produced either using technology or is neat and well-detailed by hand.	4
3.	Explain why the roller coaster meets the first three criteria.	4
4.	Calculus is used to demonstrate that graph of the roller coaster is continuous everywhere on its domain.	4
5.	The problem is well-organized and easy to follow. Computations are presented neatly by hand or typed.	4
Participation Multiplier (full=1, some =0.5, very little=0)		

Sample Roller Coaster - with different domain and range

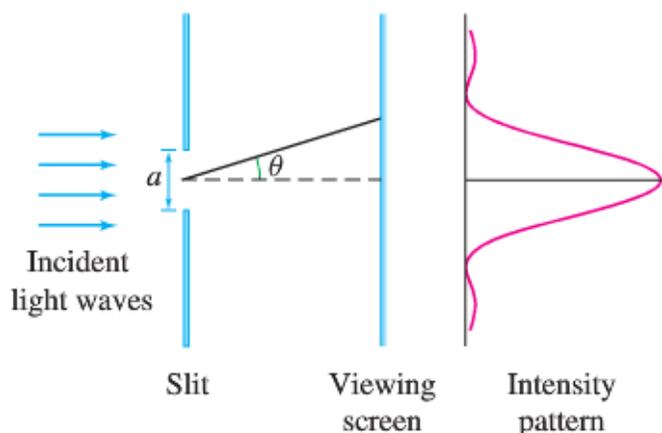


$$r(x) = \begin{cases} x & 0 \leq x < 50 \\ 10 \sin\left(\frac{x-50}{10}\right) + 50 & 50 \leq x < 50 + 35\pi \\ 40 & 50 + 35\pi \leq x < 160 \\ -\left(\frac{x-160}{4}\right)^2 + 40 & 160 \leq x < 180 \\ -2.5 \ln(x - 179) + 15 & 180 \leq x \leq 200 \end{cases}$$

**Problem 2.** Light waves of frequency  $\lambda$  passing through a slit of width  $a$  produce a Fraunhofer diffraction pattern of light and dark fringes (see figure). The intensity as a function of the angle  $\theta$  is

$$I(\theta) = I_m \left( \frac{\sin(R \sin \theta)}{R \sin \theta} \right)^2$$

where  $R = \pi a/\lambda$  and  $I_m$  is a constant.



Explain why the intensity function is not defined at  $\theta = 0$ .

Choose any two positive values for  $R$  and complete the chart.

$R = \underline{\hspace{2cm}}$

$\theta$	$\pm 1$	$\pm .1$	$\pm .01$	$\pm .001$	$\pm .0001$
$\frac{I(\theta)}{I_m}$					

$R = \underline{\hspace{2cm}}$

$\theta$	$\pm 1$	$\pm .1$	$\pm .01$	$\pm .001$	$\pm .0001$
$\frac{I(\theta)}{I_m}$					

Use calculus and the charts above to verify that  $I(\theta)$  approaches  $I_m$  as  $\theta$  approaches 0.

Grading on Problem 2 will be on the following criteria:

Item	Criteria	Points
1.	Explanation for why intensity function is not defined is clear and mathematically sound.	4
2.	$R$ is indicated and charts are filled out accurately.	4
3.	Calculus ideas are used to explained the limit of $I(\theta)$ as $\theta$ approaches 0.	4
4.	The problem is well-organized and easy to follow. Computations are presented neatly by hand or typed.	4
Participation Multiplier (full=1, some =0.5, very little=0)		