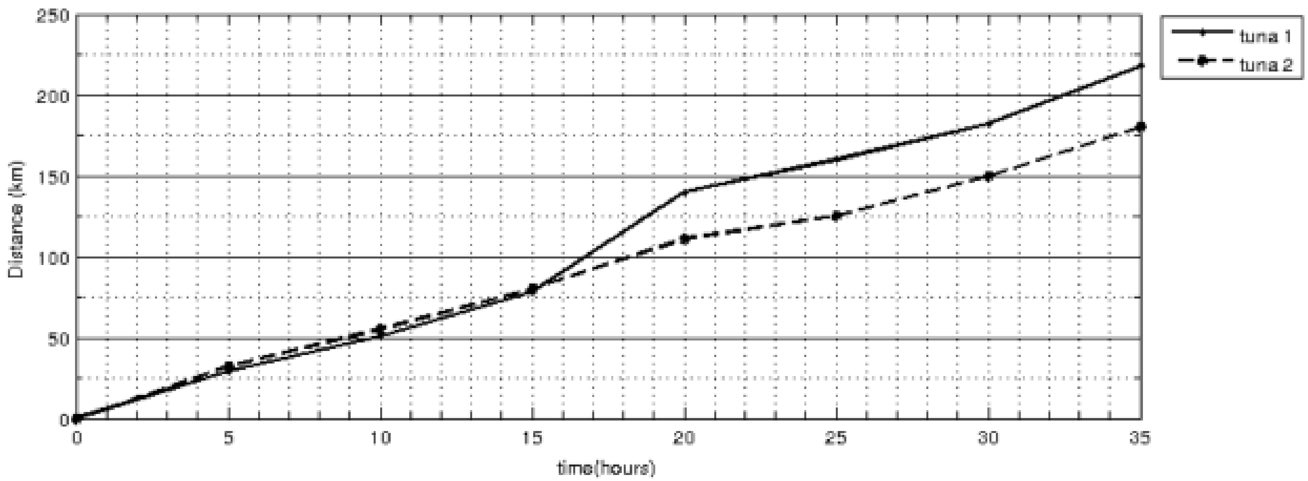
M171 Fall 2018: Second Examination Practice Problems.

You can check your answers in WebWork. Full solutions in WW available Monday evening.

Problem 1. Use the tuna swimming data (figure below) to answer the following questions

a. Determine the average velocity of each of these two fish over the 35 hours shown in the figure.

b. What is the fastest average velocity shown in this figure, and over what time interval and for which fish did it occur?



Problem 2. Find the average rate of change of the given function over the given interval. Express your answers in terms of square roots and π, do not give decimal expressions.

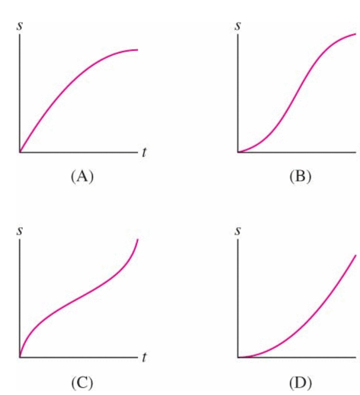
a. $\sin(x)$ over $0 \le x \le \pi/4$

b. $\cos(x)$ over $\pi/6 \le x \le \pi/3$

c. Is there an interval over which the functions $\sin(x)$ and $\cos(x)$ have the same average rate of change that is non-zero? (Hint: Consider the graphs of these functions over one whole cycle, e.g. for $0 \le x \le 2\pi$. Where do they intersect?)

Problem 3.

The graphs in the figure represent the positions s of moving particles as functions of time t. Match each graph with a description:



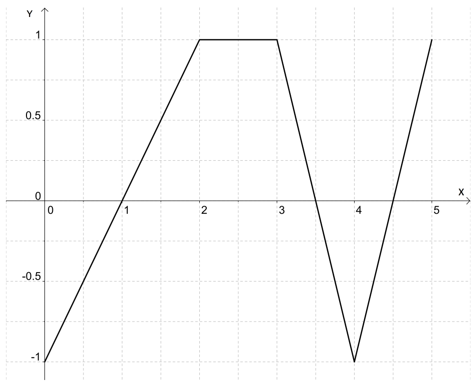
a. Speeding up

b. Speeding up and then slowing down

c. Slowing down

d. Slowing down and then speeding up

Problem 4. Consider the graph below of the function $f(x)$ on the interval $[0, 5]$.



1. For which x values would the derivative $f'(x)$ not be defined?
2. Sketch the graph of the derivative function f'.

Problem 5. Given $f(x) = \dfrac{1}{3x+1}$, find $f'(1)$ using the **limit definition** of the derivative.

Problem 6. Use the **limit definition** of the derivative to find the derivative of $f(x) = \sqrt{2x+5}$

Problem 7. What does the limit $\displaystyle\lim_{x\to 2} \frac{x^{-2} - \frac{1}{4}}{x-2}$ represent?

A) $\left.\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right)\right|_{x=2}$

B) 0

C) $\ln(2)$

D) $-\dfrac{1}{2}$

E) The limit does not exist.

Problem 8. The Monod Growth function is given by

$$G(S) = \mu \frac{S}{K+S}$$

where μ and K are constant.

A) Find $G'(S)$.

B) Find the equation of the line tangent to the curve $y = G(S)$ at $S = 0$.

C) Find the equation of the line tangent to the curve $y = G(S)$ at $S = K$

D) Which slope is larger?

Problem 9.

A) Find the equation of the line tangent to the curve $y = \frac{x}{x+1}$ at $x = 0$.

B) Find an equation of the tangent line to $y = 2e^x$ at $x = 0$ and another tangent line at $x = \ln(2)$.

Problem 10. If a fish is swimming at speed v against a current with speed c ($c < v$) the amount of energy required to swim a fixed distance L is given by

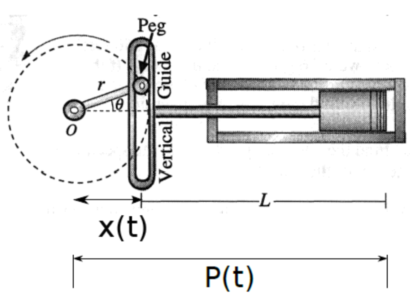
$$E(v) = \frac{aLv^2}{v-c}$$

Find the rate change of Energy with respect to speed v.

Problem 11. Suppose that $f(2) = 3$, $g(2) = 2$, $f'(2) = -2$, $g'(2) = 4$ and $f'(16) = 0$. For the following functions, find $h'(2)$.

1. $h(x) = 5f(x) + 2g(x)$
2. $h(x) = f(x)g(x)$
3. $h(x) = \dfrac{g(x)}{1 + f(x)}$
4. $h(x) = \sqrt{[f(x)]^2 + 7}$.
5. $h(x) = f(x^3 \cdot g(x))$.

Problem 12. A peg on the end of a rotating crank slides freely in the vertical guide shown in the diagram. The guide is rigidly connected to a piston which moves horizontally. The crank rotates at a constant angular speed ω in a circle of radius r.



It is easy to show that the position of the piston is given by

$$P(t) = L + r\cos(\omega t),$$

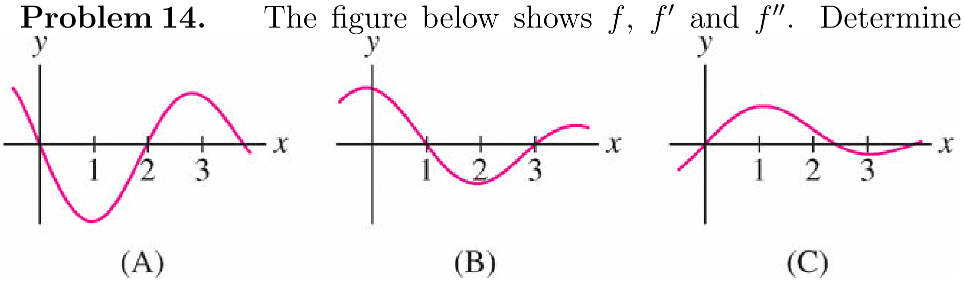
since $x(t) = r\cos(\omega t)$. Find the velocity and acceleration of the piston: $\dfrac{d}{dt}P(t)$ and $\dfrac{d^2}{dt^2}P(t)$.

Problem 13. Let

$$g(t) = \begin{cases} t^2 + b & \text{if } t \le 0 \\ at + 4 & \text{if } t > 0 \end{cases}.$$

Find a and b so that g is differentiable at $t = 0$.

Problem 14. The figure below shows f, f' and f''. Determine which is which.



Problem 15. What is the 2016$^{\text{th}}$ derivative of $f(x) = x^{2015} + e^x + \sin(x)$?

Problem 16. Find the equation of the line tangent to the curve $y = A\tan(\omega x)$ at $x = \dfrac{\pi}{3\omega}$. Here A and ω are constants.

Problem 17. Find the derivative of y with respect to x:

1. $e^{(xy)} = \cos(y^4)$.
2. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = \pi^{\frac{2}{3}}$.

Problem 18. Find an equation of the tangent line to the curve

$$x^2 + \sin(y) = xy^2 + 1$$

at the point $(1, 0)$.

Problem 19. The Lambert function, $W(x)$, is implicitly defined by the following equation:

$$W(x)e^{W(x)} = x.$$

Use implicit differentiation to find a formula for $\frac{d}{dx}W(x)$. Show that W' is given by $W' = \frac{1}{e^W + x}$.

Problem 20. Find y' using logarithmic differentiation of $y = x^{\cos(x)}$.

Problem 21. Compute the first derivative of each of the following functions:

1. $f(x) = \cos(4\pi x^3) + \sin(3x + 2)$
2. $b(t) = t^4 \cos(3t^2)$
3. $y(\theta) = e^{\sec(2\theta)}$
4. $k(s) = \ln(7s^2 + \sin(s) + 1)$
5. $u(x) = (\sin^{-1}(2x))^2$
6. $h(x) = \dfrac{8x^2 - 7x + 3}{\cos(2x)}$

Problem 22. Compute the first derivative of each of the following functions:

7. $m(x) = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}}$
8. $q(x) = \dfrac{e^x}{1 + x^2}$
9. $n(x) = \cosh(\tan(x))$
10. $w(x) = \arctan(x^2 + 1)$
11. $w(x) = x^{(x^2+1)}$
12. $x(t) = Ae^{-kt}\cos(\omega t + b)$

Problem 23. Compute the first derivative of each of the following functions:

13. $P(t) = 2^t$
14. $E(y) = y\ln(1 + e^{y^2})$
15. $l(w) = \sinh(\cos(w))$
16. $k(x) = \ln(7x^2 + 2x + 1)$
17. $q(x) = \dfrac{\ln(x)}{1 + x}$
18. $g(t) = \cosh(at^2 + b)$
19. $g(t) = \sinh(4t + 2)$

<u>Galileo's Equations</u>: $s(t) = s_0 + v_0 t - \frac{1}{2}gt^2$ $\qquad v(t) = v_0 - gt$

Problem 24. A hot air balloon ascends into the air at a constant speed of 5.0 m/s. At some point above the ground, a sand bag is released from the balloon. If it takes the sand bag 2.5 seconds to strike the ground, what was the height of the balloon at the time the sandbag was released?

Problem 25. An astronaut on the moon throws a wrench straight up at 4.0 m/s. Three seconds later it will fall downward at a velocity of 0.80 m/s.

(a) What was the acceleration of the wrench after it left the astronauts hand?

(b) How high above the point from which it was released was the wrench at 3.0 s?

(c) How long would it take the wrench to return to the position from which it was thrown?

Problem 26. A particle is moving along a line so that at time t seconds, the particle is $s(t) = \frac{1}{3}t^3 - t^2 - 8t$ meters to the right of the origin.

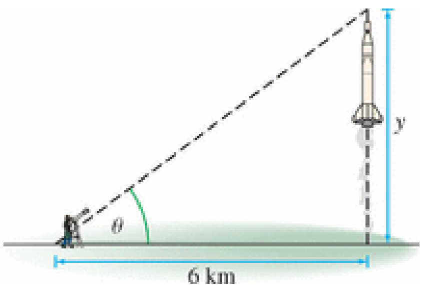
(a) Find the time interval(s) when the particle's velocity is negative.

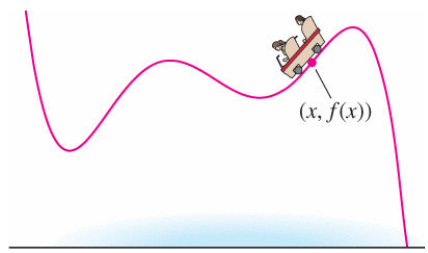
(b) Find the time(s) when the velocity is zero.

(c) Find the time interval(s) when the particle's acceleration is positive.

(d) Find the time interval(s) when the particle is speeding up. Hint: What do we need to know about velocity and acceleration in order to know that the derivative of the speed is positive?

Problem 27. A spy uses a telescope to track a rocket launched vertically from a launching pad 6 km away, as in the Figure below. At a certain moment, he angle between the telescope and the ground is equal to $\frac{\pi}{4}$ and is changing at a rate of 1 rad/min. What is the rocket's velocity at that moment?



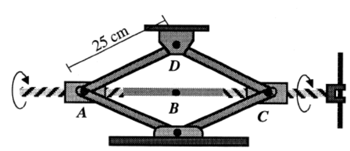


Problem 28. A roller coaster has the shape of the graph in the Figure above. Show that when the roller coaster passes the point $(x, f(x))$, the vertical velocity of the roller coaster is equal to $f'(x)$ times its horizontal velocity, $x'(t)$.

Problem 29. The height of the automobile jack shown is controlled by rotating a screw ABC which is single-threaded at each end, the pitch is 2.5 mm. If the screw ABC is rotated at a rate of 10 rpm clockwise at C, then one can show that the length of AB DECREASES at the constant rate of 2.5 cm/min, i.e., $dx/dt = -2.5$. (B is the midpoint of AC)

(a) When the length AB is 20 cm, at what rate is the length BD changing? [i.e when $x = 20$ what is dy/dt]? (Note: $25^2 = 20^2 + 15^2$).

(b) When the length AB is 20 cm, what is acceleration of the length BD assuming that x is decreasing at a constant rate, [i.e, find d^2y/dt^2].



(c) When the length AB is 20 cm, at what rate would a car sitting on the jack be moving upward? (Hint The height of the car is a constant plus $2y$)

Problem 30. As Claudia walks away from a 264-cm lamppost, the tip of her shadow moves twice as fast as she does. What is Claudias height?