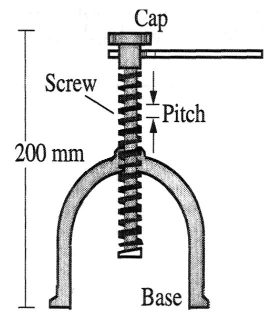
M171 S2018: First Examination Practice Problems.

Problem 1. Determine whether there exists a constant c such that the line $x + cy = 1$

- Has slope 4
- Passes through $(3, 1)$
- Is horizontal
- Is vertical

And if so find the corresponding value of c for each case.

Problem 2. Each time the jack undergoes a complete counter clockwise revolution, the threaded screw advances 2 millimeters upward. The distance between the threads of a screw is called the pitch. If the tire jack has an initial height of 200 millimeters, find the height, $h(x)$, after x turns counter clockwise in millimeters.



Problem 3. Complete the square and find the minimum or maximum value of the quadratic function

$$y = 2hx - 2hx^2.$$

Problem 4. If the alleles A and B of the cystic fibrosis gene occur in a population with frequencies p and $1-p$ (where p is a fraction between 0 and 1), then the frequency of heterozygous carriers (carriers with both alleles) is $2p(1-p)$. Which value of p gives the largest frequency of heterozygous carriers?

Problem 5. Find all values of c so that $f(x) = \dfrac{x}{x^2 + cx + 4}$ has as its domain all real numbers.

Problem 6. Given $f(x) = \ln(x)$, $g(x) = x^2 + 3$ and $h(x) = \sin(x)$ find $f \circ g \circ h$: $(f \circ g \circ h)(x) =$

Problem 7. (a) Find all angles between 0 and 2π for which $\sin(\theta) = 1/2$.
(b) Show that if $\tan(\theta) = c$ and $0 \le \theta \le \pi/2$, then $\cos(\theta) = 1/\sqrt{1 + c^2}$.

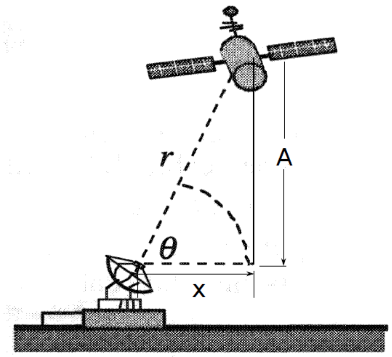
Problem 8. Simplify the following leaving no answer in terms of a trigonometric function.

a. $\sin^{-1}\left(\sin(\frac{2\pi}{3})\right)$

b. $\tan\left(\sin^{-1}(x)\right)$

c. $\sin^2\left(\cos^{-1}(x)\right)$

Problem 9.

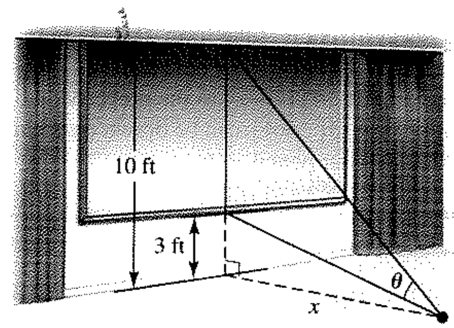


The radar tracks the satellite and computes the distance r, the angle θ.

A. Find the altitude A as a function of ONLY θ and r.

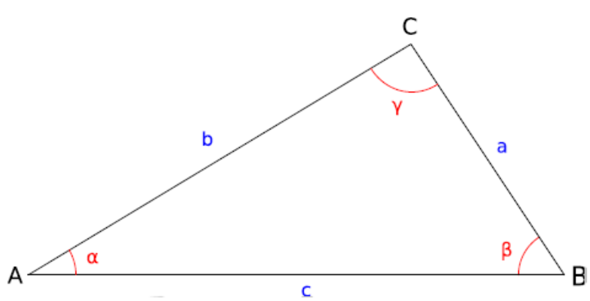
B. Find x as a function of ONLY θ and r.

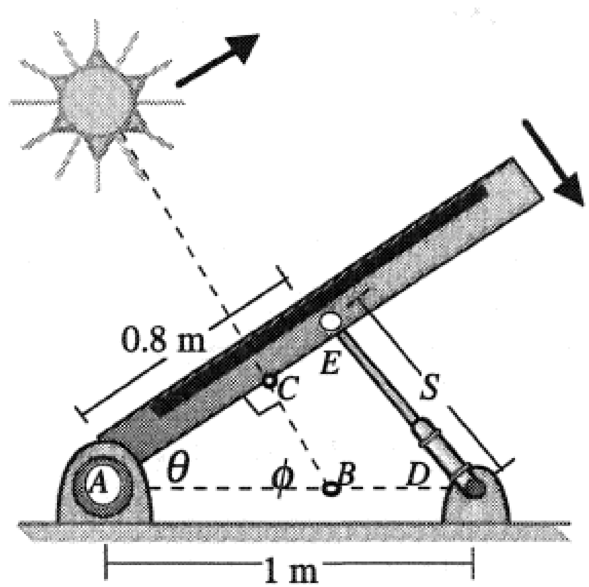
Problem 10. An auditorium with a flat floor has a large flat panel television on one wall. The lower edge of the television is 3 ft above the floor, and the upper edge is 10 ft above the floor. Express θ in terms of x.



Problem 11.

Law of cosines : $c^2 = a^2 + b^2 - 2ab\cos(\gamma)$ or
$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$ or
$b^2 = a^2 + c^2 - 2ac\cos(\beta)$.





The efficiency of a solar panel can be increased by 'tracking the sun' much as the leaves of green plants do. The solar array shown in the illustration is kept perpendicular to the rays of sun by means of a hydraulic jack DE.

To what length S must the piston be extended to keep the array perpendicular to the sun's rays when the sun is at an angle of $\phi = 45$ degrees above the horizon?

Problem 12. In 1941 British physicist G.I. Taylor noted that the radius R of the blast of a nuclear explosion should initially depend only on the energy E of the explosion, the time t after the detonation, and the density of the air. The only number having dimensions of length that can be constructed from these quantities is:

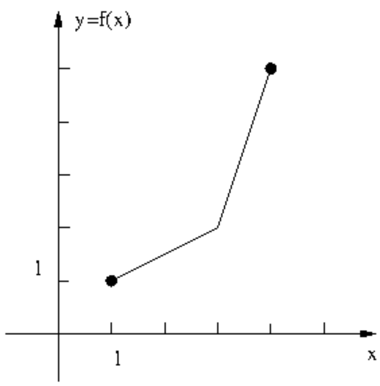
$$R = S\left(\frac{Et^2}{\rho}\right)^{\frac{1}{5}}$$

where S is a dimensionless parameter.

Assuming the above function for the radius of the blast as a function of time t, $R(t)$, find the inverse function function $t(R)$ and explain what it means. Be sure to give the domain of the inverse function.

Problem 13. Show that $f(x) = \dfrac{x}{x-1}$ is its own inverse by showing that $f(f(x)) = x$.

Problem 14. Consider the function whose graph appears below.



1. $f^{-1}(2) =$ ______
2. $f^{-1}(f(2)) =$ ______
3. Sketch the graph of f^{-1} on the same axis.

Problem 15.

a. Express the quantity as a single logarithm: $5\ln(x^{1/2}) + \ln(9x)$.

b. Solve the equation for x: $\ln(x^4) - \ln(x^2) = 9$

c. Calculate $\ln(e^4) - \ln(e^{-2})$.

Problem 16. Recall that $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$, simplify $\cosh(\ln(2))$.

Problem 17. Modern climbers use GPS to determine altitudes, but back in the day, they used altimeters, which were based on air pressure. The basic formula is

$$p = p_0 e^{kz}$$

where p_0 is the atmospheric pressure at sea level, p is the atmospheric pressure at the measurement height z and the constant k depends on temperature, the acceleration due to gravity, the universal gas constant among other things. Find a formula for the height above sea level z in terms of air pressure p and the other parameters p_0 and k.

Problem 18.

a. Solve the equation $6e^{4t} = 3$ for t.

b. Given $\log_{10} E = 4 + M$, express E as a function of M.

c. The population of a city is given by (in millions) at time t (years) is $P(t) = 2.1e^{0.1t}$, where $t = 0$ is the year 2000. When will the population double from its size at $t = 0$?

Problem 19. The velocity of a skydiver (in m/s) t seconds after jumping from a plane is $v(t) = \dfrac{a(1 - e^{-kt/60})}{k}$ where $k > 0$ and $a > 0$ are physical constants. The terminal velocity of the skydiver is the value of $v(t)$ approaches as t becomes large. Find the terminal velocity (v_T).