Math 242. Fall 2010 Name _____
Exam on Chapter 1 of *Proof*.

1. (4 pts) Write out the pronunciation of "$\{x \mid x \leq 10\}$."

2. (4 pts) These might have grammatical mistakes, given our conventions. If they do not, say "okay." If they do, note what is wrong with them.
a) $5 \subset [1, 8]$ b) $x \in S \Rightarrow x \in T$. c) $\{0, 5\} \cap (1, \infty)$ d) $3x < 6 \Rightarrow \{x \mid x < 2\}$

3. (8 pts) Here is a form: $H \Rightarrow C$.
a&b) Give two other forms that are logically equivalent to it.

c) Give its converse.

d) Give its negation.

4. (5 pts) There are five logical connectives and five corresponding set-theory terms. List the five connectives along with the corresponding set-theory term of each.

5. (4 pts) Complete the definitions (symbolically, in sentence-form, not just in English):
a) *set intersection*

b) *set equality (=)*

6. (4 pts) a) Define *tautology*.

b) State one.

7. (3 pts) Give $\{4, 6\}^c$.

8. (6 pts) Are these pairs of sentences equivalent? If so, just say "Yes". If not, say "No" and give a **specific** counterexample. [Assign particular values to each letter.]

a) $x^2 > 4x$ $\quad\quad$ $x > 4$.

b) $x < 2$ $\quad\quad$ $y < 2$

c) not($x < -5$ or $x > 7$), $\quad\quad$ $x \geq -5$ and $x \leq 7$.

9. (12 pts) Rewrite each statement using the logical equivalence that seems most appropriate. Also, cite our name for the equivalence you use.

a) ($x \geq 0$ and $x^2 > 16$) => $g(x) > 3$.

b) ($x > 7$ or $x < 0$) => $f(x) \geq 5$.

c) $x > 1$ or $f(x) \leq 7$.

d) If $f(x) > 5$, then $x < 2$ or $x \geq 10$.

e) If $f(x) > 5$, then $x < 2$ or $x \geq 10$. [This is (d) again. Do it again, another way.]

f) If $S \cap T = \varnothing$, then ($x \in S$ => $x \notin T$).

10. (4 pts) Mathematicians define "H => C" to be true whenever "H" is false. Why? Explain this as if to a student who does not know. [A few correct statements with a few of the right words will not get full credit unless it is a good **explanation**.]

11. (4 pts) We showed "$S \subset T$ or $S \subset T^c$ " is false, with a counterexample. For this problem, give, instead,
a) a clearly-labeled Venn diagram which illustrates why it is false.

b) a clearly labeled number-line picture which illustrates why it is false.

12. (6 pts) Create a truth table, with all appropriate columns (Do not skip columns!), for determining if "(not A) => B " is logically equivalent to "A or not B ." At the end, say if they are or are not logically equivalent, and why.

13. (4 pts) True or false? (For each, circle "T" or "F". No reason required.)

a) T F A proof of "$A \Rightarrow C$" proves "A and $B \Rightarrow C$ ".

b) T F A proof of "(A or B) $\Rightarrow C$ " proves "$A \Rightarrow C$ ".

c) T F $x > 5 \Rightarrow x \geq 5$.

d) T F $n > 5 \Rightarrow n \geq 6$.

14. (6 pts) If you know the following conditional is true, state what you can deduce with the given additional fact.
If a set is virid, then it is exud.

a) S is exud. Deduce:

b) S is not exud. Deduce:

c) S not virud. Deduce:

15. (10 pts) **Instructions**: Simply determine which conjectures **follow logically** (FL) from Assertion 1, or not (N). Assertion 1: If $x > 7$, then $f(x) \geq 4$.

a) FL N $f(4) \geq 5$

b) FL N If $f(x) < 3$, then $x < 8$.

c) FL N If $f(x) < 8$, then $x \leq 7$.

d) FL N If $|x| > 8$, then $f(x) > 1$.

e) FL N If $|f(x)| < 2$, then $x < 8$.

16. (8 pts) Suppose this is a fact: "If $x < 5$ and $x > 2$, then $f(x) < 8$."
If the following is also a fact, what can be deduced from those two facts?

a) $x < 4$ and $x > 3$.

b) $x < 5$ and $f(x) = 10$.

c) $f(x) > 12$.

d) $f(x) = 8$ and $x > 3$.

17. (8 pts) Suppose this is true: "If a tryk is walid and not jyd, then it is pred."
What follows logically from these additional facts?

a) The tryk is jyd and not pred.

b) The tryk is walid and not pred.

c) The tryk is not pred and not jyd.

d) The tryk is walid or not jyd.