Math 242. Exam on Chapters 1 and 2 of *Proof*. Name:_____

1. (4 pts) Complete these logical equivalences the way they were completed as theorems in the text.

a) $A \Rightarrow (B \Rightarrow C)$ is logically equivalent to

b) (A or B) $\Rightarrow C$ is logically equivalent to

2. (15 pts) Give the **negation (in positive form)** of

a) [Let S and T be given.] If $x \in S$ and $x < 9$, then $x \in T$.

b) [Let f, a, and L be given.]
For $\varepsilon > 0$ there exists $\delta > 0$ such that $0 < |x - a| < \delta$ implies $|f(x) - L| < \varepsilon$.

c) [Let $\{a_n\}$ be given.] For any m, $a_n > m$ when $n > n^*$, for some n^*.

d) [Let f be given] No horizontal line intersects the graph of f twice or more.

e) [About several piles of chips] At least two piles have at least 20 chips.

3. (6 pts) Define $x\sim$ this way: Let $x\sim = 3x$ if $x < 5$ and $x\sim = x/4$ if $x \geq 5$.
Solve for x: $x\sim = (2\sim)(8\sim)$.

4. (6 pts) There are only three types of sentences with one variable. Name them (as in Section 2.1), and for each, give one good example. [Make sure it is clear which example goes with which type.]

5. (5 pts) Solve for b: $c^2 = a^2 + b^2 - 2ab \cos C$. [Just solve it; do not simplify.]

6. (5 pts) Read this theorem and use it to do the problem:
Theorem: The graph of $x^3 - kx + c$ has local extrema where $x^2 = k/3$.
Problem: Find the value(s) of x such that $x^3 - 6x + 5$ has a local extremum there.

7. (5 pts) Read and use this theorem to do the problem.
Theorem: The sum of the two roots of $ax^2 + bx + c$ is $-b/a$.
Problem: Suppose one root of $2x^2 - 8x + k$ is -5. Find the other root.

8. (8 pts) True or false? [No reason required here, but see the next problem.]
a) T F For all x in [3, 5) there exists y in [3, 5) such that $y < x$.

b) T F For all x in [3, 5) there exists y in [3, 5) such that $y > x$.

c) T F There exists y in [3, 5) such that for all x in [3, 5), $y \le x$.

d) T F There exists y in [3, 5) such that for all x in [3, 5), $y \ge x$.

9. (3 pts) At least one of the above is false. Select one (be clear which one) and prove it is false.

10. (12 pts) Suppose this is true: If $x < 4$, then $f(x) \le 8$. Which of these follow logically (FL)?
a) FL not FL $x > 3$ whenever $f(x) > 9$.

b) FL not FL If $x \le 4$, then $f(x) \le 8$.

c) FL not FL If $x < 4$, then $f(x) < 8$.

d) FL not FL $f(x) < 10$ when $x < 3$.

e) FL not FL If $|f(x)| > 10$, then $x \ne 2$

f) FL not FL If $|x| < 3$, then $f(x) < 12$.

11. (6 pts) Suppose this is true. "If $x > 3$ and $y < 4$, then $f(x, y) < 7$."
What follows logically, if anything, from these additional facts?

a) $f(4, y) = 12$

b) $x = 5$ or $y = 2$

c) $y < 4$ [Give a deduction with "or" in it.]

12. (4 pts) Suppose this is true."If $g(x, y) > 10$, then $x \leq 2$ or $x > 15$."
What follows logically, if anything, from these additional facts?

a) $g(x, y) > 13$ and $x < 13$.

b) $3 < x < 5$.

13. (6 pts) Give the **sentence-form** definition of these:
a) [Let f be a function defined on all real numbers.] increasing function.

b) upper bound

14. (3 pts) Define *generalization*.

15. (Short essay, 4 pts) A friend of yours might think the negation of "all are" is "all are not". Explain why that is incorrect. [Be clear and convincing, as if you were explaining to someone who does not know. This can be short, but will be graded on clarity, not just on whether it is correct.]

16. (3 pts) [Note: "sup S " is a number.] Suppose we want to prove this theorem:
Theorem: "If S is infinite and bounded, then sup S is in S or there is a sequence $\{a_n\}$ such that $a_n \in S$ for all n and $a_n \to \sup S$."
Use "*Or* in the conclusion" to restate this theorem in a logically equivalent version.[Stop when you have stated the logically equivalent sentence.]

17. (5 pts) Conjecture: If $a < k$ for all $k > b$, then $a \leq b$.
Resolve it. That is, decide if it is true or false and then prove it is true or prove it is false.