Math 242. Final Exam on *Proof*.
Sections 1.1-5.2. Fall 2010

Name:__________________
initial each sheet of extra paper

1. (6 pts) Give the sentence-form definition of
a) *bounded*.

b) *onto*

2. (6 pts) These might have grammatical mistakes or unconventional usages. Which ones? Give a short, specific, indication of the error.
a) 7 is in the power set of S b) If $\{x \mid x/2 > 3\}$, then $x > 6$. c) $2x = 18 = x = 9$

3. (4 pts) Write out the pronunciation of "$\{x \mid 3x > 6\}$."

4. (6 pts) Do the two sentences have the same meaning? Yes (Y) or No (N)

a) Y N	Let $f(x) = 2x$, for all x.	Let $f(c) = 2c$, for all c.
b) Y N	Let $f(x) = x^2$, for all x.	Let $g(x) = x^2$, for all x.
c) Y N	$2x + 7 = 19$	$2y + 7 = 19$.

5. (6 pts) Some mathematical sentences are said to be "open."
a) What makes a sentence open? Give a good example.
b) What other types of sentences with a single variable are there? Give good examples.

6. (4 pts) a) Give a mathematical sentence which is a tautology.

b) Give a mathematical sentence which is always true but not a tautology.

7. (14 pts) True or False [No reason required.]

a) T F If $a^2 < b^2$ and $b > 0$, then $a < b$.

b) T F If $C \Rightarrow$ not D, then $D \Rightarrow$ not C.

c) T F If $x \in [4, 7)$ there is $y \in [4, 7)$ such that $y > x$.

d) T F If $x \in [4, 7)$ there is $y \in [4, 7)$ such that $y < x$.

e) T F There is $y \in [4, 7)$ such that if $x \in [4, 7)$, $y \geq x$.

f) T F If $|x - a| < 1$, then $x > a - 1$.

g) T F If $f(x) \leq g(x)$ for all x, then $\sup\{f(x)\} \leq \inf\{g(x)\}$.

8. (15 pts) Suppose this is given: If $x \leq 4$, then $f(x) > 6$.
Determine which of these follow logically.

a) FL not FL If $x < 4$, then $f(x) > 5$.

b) FL not FL If $f(x) < 8$, then $x > 4$.

c) FL not FL If $x < 5$, then $f(x) > 3$

d) FL not FL If $x \leq 4$ and $y > 7$, then $f(x) > 6$.

e) FL not FL If $x \leq 4$, then $f(x) > 6$ or $f(x) < 2$.

9. (9 pts) Suppose this is given: If $x > 7$ or $x < 2$, then $g(x) \leq 4$.
Determine which of these follow logically.

a) FL not FL If $x > 8$, then $g(x) < 5$.

b) FL not FL If $g(x) > 6$, then $x \leq 7$.

c) FL not FL If $x \geq 2$ and $g(x) > 5$, then $x \leq 7$.

10. (6 pts) Read and use this result, assumed true for this problem (and only this problem):
$n^* = (n - 1)/2$ if n is odd and $n^* = n/2 + 3$ if n is even.
Solve for n: $n^* = (9^*)(8^*)$.

11. (16 pts) What can be deduced from the Given Result and the additional fact?
Given Result: Grigs that are zap and trib are rad or yat.
a) The grig is zap and trib and not yat.

b) The zap grig is not rad and not yat.

c) The grig is neither rad nor yat.

d) The trib grig is not yat. [Deduce something non-trivial.]

12. (16 pts) Give the **negation (in positive form)** of
a) [g is given] $g(x) < 7$ for all $x > 5$.

b) [f is given.] For $\varepsilon > 0$ there exists $\delta > 0$ such that $|f(x) - f(t)| < \varepsilon$ whenever $|x - t| < \delta$.

c) If c is greater than 0, then the graph of $x^2 + bx + c$ is entirely above the x-axis.

d) [about several piles of chips] At least one pile has at least 5 chips.

13. (4 pts) State the logical equivalence we know as A Hypothesis in the Conclusion.

14. (12 pts) Make a truth table, with **all** relevant columns, to determine if "A or not B" is logically equivalent to "$B \Rightarrow A$." [As always, put the column for A on the left.]

15. (6 pts) Conjecture: If $g \circ f$ is onto, then f is onto.

This is false. Sketch the **simplest counterexample** that can be given by a picture of the type with A and B and C represented as circles (like the kind where f and g connected dots within them), with the relevant parts identified.

^^^^^^^^^^^^^^^ **Proofs and Resolutions**

Instructions: **Read These!**

- **Do the work again here!** If a result you want to use is similar to these and reminds you of a theorem or example done in class or the book or homework, **do the work again here**.
- **Cite reasons for each step**. Key prior results **must** be cited when used. Hypotheses **must** be cited when used. Do not cite very similar results to "prove" things. If something you want to use is distinctly prior, cite it and just use it. Your proofs must use prior results, that is, results prior at the time the type of problem was first encountered.
- Problems will be graded on the use of logic and organization, as well as ideas.
- Counterexamples must be **complete**.
- Do your work on the blank sheets provided. Initial each sheet.

(10 pts each)

16. Resolve this Conjecture: **P**(R) $\subset$ **P**(S), then $R \subset S$. [**P** is a notation for power set.]

17. Prove **by induction**: $\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

18. Resolve the Conjecture: If x is rational and y is irrational, then $x - y$ is irrational.

19. Recall that sup S is the least upper bound of S. Assume each of S and T is non-empty and bounded above. Prove: If $S \subset T$, then $\sup S \leq \sup T$. [Do not make assertions without clearly referencing the reason for each.]

20. Prove: For $m > 0$ there exists k such that $\ln x > m$ if $x > k$.

21. Definition: p is an interior point of S iff there exists $d > 0$ such that $(p-d, p+d) \subset S$.

Conjecture: If p is an interior point of S and p is an interior point of T, then p is an interior point of $S \cap T$.

22. Prove: If, for all sets S and T, $f(S \cap T) = f(S) \cap f(T)$, then f is one-to-one.