1) Suppose that the position of an object at time $t$ is given by
\[ \mathbf{r}(t) = (\cos t, \sin t, t^2 + \frac{3}{2}t^\frac{3}{2}) \]. Find:

a) The velocity at time $t$
\[ \mathbf{v}(t) = \mathbf{r}'(t) = (-\sin t, \cos t, 3t^\frac{3}{2}) \]

b) The speed at time $t$
\[ \text{speed} = |\mathbf{v}(t)| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (3t^\frac{3}{2})^2} = \sqrt{\sin^2 t + \cos^2 t + \frac{9}{4}t^2} = \sqrt{1 + \frac{9}{4}t} \]

(c) The acceleration at time $t$
\[ \mathbf{a}(t) = \mathbf{v}'(t) = (-\cos t, -\sin t, \frac{3}{4}t^\frac{3}{2}) \]

(c) The length of the path of motion between $t=0$ and $t=\frac{4}{3}$.
\[ L = \int_0^{\frac{4}{3}} |\mathbf{r}'(t)| \, dt = \int_0^{\frac{4}{3}} \sqrt{1 + \frac{9}{4}t} \, dt = \left[ \frac{2}{9}(1 + \frac{9}{4}t)^{\frac{3}{2}} \right]_0^{\frac{4}{3}} = \frac{57}{27} \approx 2.11 \\
= \left(\frac{8}{27}\right)(4)^{\frac{3}{2}} - \frac{8}{27}(1)^{\frac{3}{2}} = \frac{57}{27} \]

2) Calculate \( \iint_D x \, dA \) where $D$ is the triangular region with vertices $(0,0)$, $(4,1)$, $(1,-1)$.

\[ \begin{aligned}
\mathbf{y} &= \mathbf{x} \\
\mathbf{z} &= 0 \\
\mathbf{w} &= 1 \\
\mathbf{v} &= -\mathbf{x} \\
\end{aligned} \]

\[ \iint_D x \, dA = \int_0^1 \int_{-x}^x x \, dy \, dx = \left[ xy \right]_0^x = \int_0^1 2x^2 \, dx = \frac{2}{3}x^3 \bigg|_0^1 = \frac{2}{3} \]

\[ \frac{2}{3} \]
3) Calculate \( \int_0^1 \int_y^{\sqrt{2-y^2}} \frac{\sqrt{2-y^2}}{x^2+y^2} \, dx \, dy \) by changing to polar coordinates.

\[ \int_0^{\sqrt{2}} \int_0^r r^2 \, r \, dr \, d\theta = \int_0^{\sqrt{2}} \left. \frac{1}{4} r^4 \right|_0^{\sqrt{2}} \, d\theta = \int_0^{\pi/4} 1 \, d\theta = \frac{\pi}{4} \]

4) a) Sketch the region of integration for \( \int_{-1}^1 \int_{x^2}^{1} (1+y^{3/2})^5 \, dy \, dx \).

b) Switch the order of integration for the integral in a).

\[ \int_0^{\sqrt{2}} \int_0^{\sqrt{y}} (1+y^{3/2})^5 \, dx \, dy \]

c) Calculate the integral in b).

\[ = \int_0^{\sqrt{2}} (1+y^{3/2})^5 \left. \frac{\sqrt{y}}{y^{3/2}} \right|_0^{\sqrt{y}} \, dy \]

\[ = \int_0^{\sqrt{2}} 2^{5/2} \left(1+y^{3/2}\right)^5 \, dy \]

\[ = \left[ \frac{2^{5/2}}{3} \left(1+y^{3/2}\right)^6 \right]_0^1 = 2^{5/2} \left(2^6 - 1\right) = \frac{2}{9} \left(64 - 1\right) = \frac{2}{9} (63) = 14 \]
5) Calculate the line integrals.

30) a) \( \int_C (x+y) \, ds \), where \( C \) is the straight line segment from \((1,2)\) to \((4,6)\).

\[
\begin{align*}
&x = 1 + 3t \\
y = 2 + 4t, \quad 0 \leq t \leq 1
\end{align*}
\]

\[
\int_0^1 (1 + 3t + 2 + 4t) \sqrt{3^2 + 4^2} \, dt
\]

\[
= \int_0^1 (3 + 7t) \sqrt{5} \, dt = \sqrt{5} \left. (3t + \frac{7}{2}t^2) \right|_0^1 = \sqrt{5} \left( \frac{15}{2} \right)
\]

b) \( \int_C x \, dy - y \, dx \), where \( C \) is the semi-circle \( x^2 + y^2 = 9, \ y \geq 0 \), oriented counterclockwise.

\[
\begin{align*}
&x = 2 \cos t, \quad 0 \leq t \leq \pi \\
y = 2 \sin t
\end{align*}
\]

\[
\int_0^\pi 2 \cos t \left( 2 \cos t \right) - \left( 2 \sin t \cdot (-2 \sin t) \right) \, dt = \int_0^\pi 4 \cos^2 t + 4 \sin^2 t \, dt
\]

\[
= \int_0^\pi 4 \, dt = 4 \pi
\]

\( \frac{\pi}{2} \)

31) \( \int_C F \cdot dr \), where \( F(x,y) = \langle 2x+y, \ x-2y \rangle \) and \( C \) is as pictured.

\[
\begin{align*}
\frac{\partial P}{\partial y} &= 1 = \frac{\partial Q}{\partial x}, \quad F \text{ is conservative.}
\end{align*}
\]

\[
\phi (x,y) = x^2 + xy - y^2 \text{ is potential}
\]

\[
\int_0^1 F \cdot dr = \int_{(2,1)}^{(0,0)} (2,1) - \phi (4,0)
\]

\[
= \sqrt{5}
\]
Set up iterated triple integrals, with appropriate limits, for finding the volume of the solid bounded by \( z = x^2 y^2 \) and \( z = 8 - x^2 - y^2 \) in:

a) rectangular coordinates

\[
\text{Vol} = \int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_x^{8-x^2-y^2} 1 \, dz \, dy \, dx
\]

b) cylindrical coordinates

\[
\text{Vol} = \int_0^{2\pi} \int_0^2 \int_0^{8-r^2} r \, dz \, dr \, d\theta
\]

6) Set up an iterated triple integral, with the appropriate limits in the coordinates of your choice, for finding the volume of the region that lies between the spheres \( x^2 + y^2 + z^2 = 4 \) and \( x^2 + y^2 + z^2 = 9 \), and "inside" the cone \( z = \sqrt{x^2 + y^2} \) (that is, \( z \geq \sqrt{x^2 + y^2} \)).

DO NOT EVALUATE.

\[
\text{Vol of } A = \int \int \int_A 1 \, dV
\]

Spherical coords

\[
\text{Vol} = \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^3 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta
\]