1) Given $f(x, y, z) = x^2 z (2y + z)^2$, calculate the gradient of $f$ at $(2, -1, 1)$ and use it to calculate the following:
   a) $\nabla f(2, -1, 1) =$

2) $\text{maximum rate of change of } f \text{ at } (2, -1, 1) =$

3) $\text{rate of change of } f \text{ at } (2, -1, 1) \text{ in the direction } <2, 1, 2> =$

4) $\text{find a direction in which the rate of change of } f \text{ at } (2, -1, 1) \text{ is zero}$

5) $\text{find an equation for the tangent plane to the surface } x^2 z (2y + z)^2 = 4 \text{ at the point } (2, -1, 1)$. 

6) If $x, y, \text{ and } z$ are related by $x^2 z (2y + z)^2 = 4$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $x = 2$, $y = -1$, and $z = 1$.

7) If $x, y, \text{ and } z$ are related by $x^2 z (2y + z)^2 = 4$, use linear approximation to find an approximate value for $z$ when $x = 2.1$ and $y = -1.2$. 
2) Below is a level curve picture for the function $f(x,y)$.

Circle the best answer:

a) $f_x(2,1)$ equals i) -2 , ii) -1 , iii) 2
b) $f_y(2,1)$ equals i) -2 , ii) -1 , iii) 1
c) $f_{yx}(2,1)$ is i) < 0 , ii) > 0 , iii) > 0
d) $f_{xy}(2,1)$ is i) < 0 , ii) > 0 , iii) > 0
e) $f_{yx}(2,1)$ is i) < 0 , ii) > 0 , iii) > 0
f) $D_{<-1,2>0}$

3) Suppose a conical cow's volume increases at the rate of $\frac{\pi}{10}$ ft$^3$/day. If the cow's height is increasing at the rate of $\frac{1}{50}$ ft/day when the cow's radius is 3 ft and height is 4 ft, at what rate is the radius changing?

$$V = \frac{1}{3} \pi r^2 h$$
4) Find all critical points of \( f(x,y) = x^2 + 3xy + y^2 - 4x - y \) and classify (rel. max, rel. min, or saddle).

5) Find the max and min of \( f(x,y) = xy \) on the ellipse \( x^2 + 2y^2 = 8 \).

Extra Credit! Estimate the max and min of \( f(x,y) = xy \) subject to \( x^2 + 2y^2 = 8 \) without explicitly solving this new problem.