2) Given \( f(x,y,z) = \sqrt{x^2 + y^2} \)

a) Find a unit vector that points in the direction in which \( f \) increases most rapidly at \( P(3,2,4) \).

b) What is the rate of change of \( f \) at \( P(3,2,4) \) in the direction found in a)?

c) Find an equation of the tangent plane to \( \sqrt{x^2 + y^2} = 5 \) at \( P(3,2,4) \).

d) Given \( \sqrt{x^2 + y^2} = 5 \), find \( \frac{dz}{dy} \) at \( P(3,2,4) \).

e) Without using a calculator, give me a good (algebraic) approximation of \( \sqrt{3^2 - (1.9)^2} \).

2) Suppose that resistance \( R \) is given by \( R = \frac{1}{R_1 + R_2} \) and that \( R_1 \) and \( R_2 \) are changing at the rates of 2 and -3 ohms per sec, respectively. At what rate is \( R \) changing when \( R_1 = 100 \) ohms and \( R_2 = 200 \) ohms?

3) The picture below is a contour (level curve) plot of a function \( z = f(x,y) \) of two variables. Assume that the distance between adjacent drawn curves is 1 unit.

a) Sketch in \( \nabla f(2,3) \), with appropriate length and direction.

b) Using a), estimate the rate of change of \( f \) at \( P(2,3) \) in the direction of \( \mathbf{v} = \langle 3, 4 \rangle \).

c) Suppose an object moves across \( P(2,3) \) with velocity \( \langle 3, 4 \rangle \). Using b), estimate the time rate of change of \( f \).

4) Find all critical points of \( f(x,y) = x^2 + 4xy + y^2 - 2x + 8y + 3 \) and classify each as being a point at which \( f \) has a relative max, min, or saddle.

5) a) Find the max and min of \( f(x,y) = x^2 + y^2 - 2x \) subject to \( x^2 + y^2 = 4 \).

b) What are the absolute max and absolute min of \( f(x,y) = 2x^2 + y^2 - 2x \) on the region \( x^2 + y^2 \leq 4 \)?