1. **Definitions.** (12 pts) Define these in sentence-form.
   a) accumulation point
   
   b) open set

2. (8 pts) Assume \( f \) is defined on \((0, \infty)\). Give the negation of \( \lim_{x \to \infty} f(x) = \infty \).

3. (25 pts) True or False, no reason required.
   a) T   F   If \( a_n < b_n < 0 \) for all \( n \) and \( \{b_n\} \) is unbounded, then \( a_n \to -\infty \)
   
   b) T   F   If \( f \) is differentiable on a closed and bounded interval, then it is uniformly continuous.
   
   c) T   F   If \( f \) is uniformly continuous on a bounded open interval, then \( f \) is bounded on that interval.
   
   d) T   F   If \( 0 < x_n < 1 \) for all \( n \), then \( (x_n)^n \to 0 \).
   
   e) T   F   If \( f' \) exists for all \( x \) and \( f'(c) > 0 \), then there is a \( \delta > 0 \) such that \( f(c) < f(x) \) for all \( x \) in \((c, c + \delta)\).

4. **Examples.** (15 pts) Give an example of each. You do not need to prove that your example works, but make sure it does! Give
   
   a) Functions such that \( \lim(xg(x)) = L \), but \( (\lim f(x))(\lim g(x)) \) is not \( L \).
   
   b) A simple function which is asymptotic to \( \frac{1}{x + x\sqrt{x + 1 + x^2}} \) as \( x \to 0^+ \).
   
   c) A non-linear uniformly continuous function which is unbounded.

5. **Counterexamples:** (12 pts) These conjectures are false. Give a counterexample.
   
   a) Conjecture: If \( f' \) exists for all \( x \), then \( f' \) is continuous.
   
   b) Conjecture: If \( f' \) exists for all \( x \) and \( f'(c) > 0 \), then there is a \( \delta > 0 \) such that \( f \) is increasing on \([c, c + \delta)\).
Proofs and disproofs. Demonstrate that you know how proofs and disproofs are written.
Do not cite similar results to “prove” these. If a result you want to use is similar to these, or equally difficult, or reminds you of a theorem or example done in class or the book or homework, do the work again here. If something you want to use is distinctly prior, use it and cite it. Don’t argue, prove.

(Do the next eight problems, 16 points each, for a total of 128 points)

6. State and prove either:
   Option A) Rolle’s Theorem or Option B) The Mean Value Theorem.

7. Prove: Let $S$ be a nonempty set which is bounded above. There exists a sequence $x_n \in S$ such that $x_n \to \sup S$.

8. Prove: If $\lim_{x \to \infty} f(x) = L$ and $x_n \to \infty$, then $\lim f(x_n) = L$.

9. Prove: If $f$ is continuous and $f(a) < k$, then there exists a neighborhood of $a$ in which $f(x) < k$.

10. Resolve this Conjecture: If $f$ and $g$ are differentiable everywhere, $f(a) = g(a)$, and $f'(x) < g'(x)$ for all $x$, then $f(x) < g(x)$ if $x > a$.

11. Prove: If $f$ is differentiable at $c$, then $f$ is continuous at $c$.

12. Prove: If $f$ is uniformly continuous and $\{x_n\}$ is Cauchy, then $\{f(x_n)\}$ is Cauchy.

13. Prove: If $f'(x) \to 4$ as $x \to 0$ and $f'(0)$ exists, then $f'(0) = 4$. [Do not assert $f'$ is continuous--that is not given.]