1 Example: Estimating the Mean Sale Price for All Used Mustang Cars

What is the average price of a used Mustang car? To answer this question, you collect a random sample of \( n = 25 \) Mustangs from a website (autotrader.com) and record the price (in $1,000’s) for each car (See Figure 1). How can we use this sample to estimate the average price for \textit{all} used Mustang cars? The sample mean of $15,980 provides a \textit{point estimate} for the parameter \( \mu = \) “mean price of all used Mustang cars”, but how close is $15,980 to \( \mu \)? To answer this question, let’s assume the population of all used Mustang cars is many, many copies of the original sample. Then simulate a distribution of sample means from \textit{resampling} with replacement from our sample of 25 cars, computing the sample mean from each resample. For example, one of our resamples may look like the sample shown in Figure 2. Note that some of the cars in the original sample were not selected for the resample, but some of the cars were selected more than once.

Figure 3 plots the sample means from 4000 resamples of the original data. Now, we can use the standard deviation of these simulated sample means (2.216) as our measure of how much sample means vary from sample to sample. We can calculate an approximate 95% confidence interval for \( \mu \) by: \( 15.98 \pm 2 \times 2.216 = 15.98 \pm 4.432 = (11.55, 20.41) \). That is, we are 95% confident that the mean price for \textit{all} used Mustang cars is between $11,550 and $20,410.
Figure 2: Bootstrap resample of 25 used Mustang cars (Lock Morgan, 2014).

Figure 3: Sample means from 4000 resamples of 25 used Mustang cars from the original sample. The top right dotplot displays the original sample. The bottom right dotplot displays the last resample. Note that the dotplot of sample means is centered close to the original sample mean of $\bar{x} = 15.98$. Why? (Lock Morgan, 2014).
2 Bootstrap Sampling Distributions

Bootstrapping is a statistical method for generating confidence intervals for a parameter by assuming our population is many many copies of the observed data, then simulating many many samples from this population by sampling with replacement from the observed data. In order to calculate a confidence interval for a parameter, e.g., a population mean, we need to know how far statistics, e.g., sample means, will vary from the parameter on average – this quantity is called the standard error. We use the bootstrapping distribution of simulated statistics to calculate the standard error, which we use in our confidence interval calculation. Since we are resampling from our original sample, this method will only work if we have an unbiased sampling method – if our sample is representative of the population.

A sampling distribution of a statistic is a probability distribution of a sample statistic. For example, the sampling distribution of a sample mean describes the variation in sample means across many samples. Imagine drawing 100,000 samples of size \( n \) from a population, calculating a sample mean from each of these samples, then plotting these sample means in a histogram – this would display a sampling distribution of the sample mean.

In practice, we only draw one random sample from the population. But we can use this one sample to generate a bootstrap sampling distribution of sample means by sampling with replacement from our sample. A bootstrap resample is the same size as the original data, and consists of data points from the original data. The only difference is that the resampling is done “with replacement” so a bootstrap resample typically contains several replicates of some values and is missing other values completely. We can repeat this process many times and store the statistics generated from each resample. The result is a bootstrap distribution (or a resampling distribution) which can be used as a replacement for the unknown sampling distribution. In particular, we can use the spread (standard error) of the bootstrapped sample statistics as a substitute for the spread (standard error) of our statistic. See Figure 4 for an illustration of this process.

3 Example: Estimating the Mean Finger Tapping Rate After Caffeine Consumption

A random sample of male college students was selected to participate in an experiment to assess the effects of caffeine on motor skills. The students were divided at random into two groups – one group drank 200 mg of caffeine and the other group drank a placebo drink. After a two hour period, each student was tested to measure finger tapping rate (number of taps in two minutes). Data from the caffeine group are pre-loaded into the Bootstrapping version of the One Mean applet at [http://www.rossmanchance.com/ISIapplets.html](http://www.rossmanchance.com/ISIapplets.html), shown in Table 1. The sample mean and sample standard deviation of tapping rates are \( \bar{x} = 474 \) and \( s = 24.647 \) with sample size \( n = 12 \).
Figure 4: Illustrated figure of the bootstrapping process from https://onlinecourses.science.psu.edu/stat555/node/119.

Table 1: Finger-tapping rates for caffeine group.

<table>
<thead>
<tr>
<th>ID</th>
<th>Tapping Rate</th>
<th>ID</th>
<th>Tapping Rate</th>
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<tbody>
<tr>
<td>1</td>
<td>446</td>
<td>7</td>
<td>455</td>
</tr>
<tr>
<td>2</td>
<td>453</td>
<td>8</td>
<td>474</td>
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<td>3</td>
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<td>5</td>
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</tr>
<tr>
<td>6</td>
<td>460</td>
<td>12</td>
<td>511</td>
</tr>
</tbody>
</table>

We would like to estimate the true finger tapping rate in the population of male college students after drinking 200 mg of caffeine. We can calculate a confidence interval for the population mean finger tapping rate using the bootstrapping method by following these steps:

1. Generate a resample of tapping rates from the data set by sampling 12 observations with replacement from the original data.
2. Calculate the sample mean tapping rate from the bootstrap resample.
3. Repeat steps 1-2 thousands of times.
4. Plot the bootstrapped sample means to display the bootstrap sampling distribution.
5. Use the standard deviation of the bootstrap sampling distribution for the standard error of the sample mean in the calculation of a confidence interval.

The output of the Bootstrapping One Mean applet is shown in Figure 5. The left plot shown in the applet output is a dotplot of the most recent bootstrap resample, also shown in the table above this plot. The right plot is a histogram of the bootstrap sampling distribution. It displays a mean of 473.918 and a standard deviation of 6.837. We can use this standard deviation to calculate an approximate 95% confidence interval for the population mean:

\[
 \bar{x} \pm 2 \times SE = 474 \pm 2(6.837) = (460.33, 487.67).
\]

Thus, we are 95% confident that the true mean tapping rate in the population of male college students two hours after drinking 200 mg of caffeine is between 460.33 and 487.67 taps per two minutes.

![Figure 5: Output of bootstrapping process with finger tapping data using Bootstrapping One Mean applet at](http://www.rossmanchance.com/ISIapplets.html)

4 Extra Resources

For an additional explanation of the bootstrapping method, watch the first five minutes of this video:

[http://www.lock5stat.com/videos/BootstrapIntro.mp4](http://www.lock5stat.com/videos/BootstrapIntro.mp4)
5 References


