1. A group has the property that the square of every element is the identity. Is the group necessarily abelian?

2. Prove that a subgroup of index 2 is normal. Show that a subgroup of index 3 need not be normal.

3. If \( \alpha \) is the root of a quadratic equation with integer coefficients, then we can define the “ring extension” 
\[
\mathbb{Z}[\alpha] = \{ m + n\alpha : m, n \in \mathbb{Z} \}.
\]
Define the two rings \( R_1 = \mathbb{Z}[^5] \) and \( R_2 = \mathbb{Z} \left[ \frac{1 + \sqrt{5}}{2} \right] \).
Show that
- \( R_1 \) is a proper subring of \( R_2 \).
- There is no ring isomorphism from \( R_1 \) onto \( R_2 \). Hint: integers cannot map to irrationals, why?)

4. Prove that if \( M \) denotes a maximal ideal in the integral domain (commutative with unit) \( R \) then the quotient ring \( R/M \) is a field.