Problem 1. Let $G$ be a multiplicative group and let $n$ be a positive integer. Show that if $G$ has at least one subgroup of order $n$, then the intersection of the family of all subgroups of $G$ having order $n$ is a normal subgroup of $G$. 
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Problem 2. Suppose that $G$ is a multiplicative group for which the quotient group $G/Z(G)$ is a cyclic group, where $Z(G)$ is the center of $G$:

$$Z(G) = \{ h \in G \mid hg = gh \text{ for every } g \in G \}.$$ 

Prove that $G$ is an abelian group.
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Problem 3. Let $p(x) = x^4 + 9x^2 - 3x + 6$. Prove that the ring $\mathbb{Z}[x]/(p(x))$ is an integral domain (i.e., has no non-zero zero divisors).
Problem 4. Let $F$ be a field. Prove that $F[x]$ principal ideal domain (i.e., every ideal of $F[x]$ is a principal ideal).