(1) In each case below, give examples of pairs of sequences \( \{x_n, y_n\} \) such that \( x_n \to 0 \) and \( y_n \to \infty \) and which satisfy the specified limits as \( n \to \infty \). Prove that your examples are really correct.
   (a) \( x_n y_n \to 0 \)
   (b) \( x_n y_n \to 6 \)
   (c) \( x_n y_n \) is bounded but has no limit.

(2) Suppose that second derivative \( f''(x) < 0 \) on \((a, b), a < b\), and the derivative \( f' \) is continuous on \([a, b]\). Prove that \( (b, f(b)) \) must lie under the tangent line to the graph at the point \((a, f(a))\).

(3) Given the sequence of functions
   \[ f_{nk}(x) = \frac{2n^k x}{(1 + n^2 x^2)^2} \quad n = 1, 2, \ldots, \]
   defined for \( 0 \leq x \leq 1 \) and \( k \in \mathbb{R} \),
   (a) for what \( k \) does \( f_{nk} \) converge uniformly on \([0, 1]\) and what is the uniform limit \( f = \lim_{n \to \infty} f_{nk} \)?
   (b) Find an antiderivative \( I_{nk}(x) = \int f_{nk}(x) \, dx \), for all \( k, n \),
   (c) For what \( k \) values does \( \lim_{n \to \infty} \int_0^1 f_{nk}(x) \, dx \) exist? Compare with the answer to Problem (3a).