

## Dynamical Systems M.S. exam 2002

1. Find bifurcation value(s) of the parameter  $r \in \mathbf{R}$  and describe the type of the bifurcation(s) for a differential equation

$$\dot{x} = x - \frac{rx}{1+x^2}.$$

Also, draw bifurcation diagram and phase portrait before, at and after the bifurcation.

2. Consider the following system of differential equations in the positive quadrant  $Q := \{(x, y) : x, y \geq 0\}$

$$\dot{x} = x(1 - x - 2y) \tag{1}$$

$$\dot{y} = y(1 - y - x/2). \tag{2}$$

- a) Find and classify all the equilibrium points.
- b) Show that there is  $l > 0$  such that  $R := [0, l] \times [0, l]$  is positively invariant (i.e. if an initial condition is in  $R$  then so is the whole solution curve for  $t \geq 0$ ).
- c) Show that there are no periodic orbits.
- d) Show that there is a heteroclinic orbit between the two non-zero equilibria.

3. Let  $f : [0, 1] \rightarrow [0, 1]$  be as depicted below.

- a) How many distinct periodic orbits of period 3 are there?
- b) Show that there exists  $x \in [0, 1]$  that is neither periodic nor eventually periodic (under  $f$ ).
- c) Show that there is  $x \in [0, 1]$  with a dense orbit in  $[0, 1]$ .