1. For a linear transformation $T : V \to V$, where $V$ is a real vector space of dimension $n$, $0 < n < \infty$, let $V_0$ denote the range of $T$ and $T_0 : V_0 \to V_0$ the restriction of $T$ to $V_0$.

(a) Is $T_0$ an isomorphism? Justify your answer.

(b) If $T_0$ is an isomorphism, and the minimal polynomial of $T$ is divisible by $\lambda^k$ then show that $k = 0$ or $k = 1$.

2. Let $(V, \langle \cdot, \cdot \rangle)$ denote a complex inner product space. Suppose the operator $T : V \to V$ is normal, with $T \neq 0$, and also has the property that $P = T^*T$ is an orthogonal projection on $V$. Show that there exists a nonzero subspace $W \subset V$, such that $\|Tw\| = \|w\|$ for each $w \in V$. (Use the spectral theorem.)

3. For parts (a) and (c) you may answer without proof.

(a) Find the jordan form of the following matrix (the field is $\mathbb{C}$).

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

(b) Find the jordan form of the following matrix (the field is $\mathbb{C}$).

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

(c) Find all possible jordan forms for a $(3\times3)$-nilpotent matrix $N$. Recall that $N$ is nilpotent if $N^k = 0$ for some positive integer $k$. 

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**MS Exam: LINEAR ALGEBRA**

**Instructions:** Attempt all problems, showing work. Note: all vector spaces are finite dimensional.