

## STAT 501 Comprehensive Exam - January 4, 2012

Note: Start each problem on a new page. You may use the equation sheet from class with Chebychev's and Jensen's Inequalities added to it.

- Approximately  $1/125$  of all births are fraternal twins and  $1/300$  are identical twins. Elvis Presley had a twin brother (who died at birth). What is the probability that Elvis was an identical twin? You may approximate the probability of a boy or girl birth as  $1/2$ .
- Let  $P(A) = P(B) = 1/3$  and  $P(A \cap B) = 1/10$ . Find
  - $P(A^c \cup B^c)$ .
  - $P(B \cap A^c)$ .
- A certain river floods every year. Suppose that the low-water mark is set at 1, and the high water mark  $Y$  has cdf
 
$$F_Y(y) = P(Y \leq y) = \begin{cases} 1 - 1/y^2, & 1 \leq y < \infty \\ 0 & \text{elsewhere} \end{cases}$$
  - Verify that  $F_Y$  is a cdf.
  - Find the pdf of  $Y$ .
  - The low water mark is reset at 0 and a unit of measurement equal to  $1/10$  of that given previously is used. Let  $Z$  denote the new high water mark. Find  $Z$  in terms of  $Y$  and find  $F_Z(z)$ .
- Let  $X|N = n$  be  $Gam(n, \beta)$  and  $N \sim Geom(p)$ .
  - Find the marginal distribution of  $X$ .
  - Find the  $E(X)$  and  $Var(X)$ .
- Let  $X \sim Gam(r, 1)$  and  $Y \sim Gam(s, 1)$  with  $X$  and  $Y$  independent. Let  $U = X + Y$  and  $V = X/U$ .
  - Find the joint distribution of  $U$  and  $V$ . Are  $U$  and  $V$  independent? Justify your answer.
  - Find the marginal distribution of  $V$ .
- For random variables  $X$  and  $Y$ , function  $g(x, y)$  and constants  $a$  and  $b$  show that if  $a \leq g(x, y) \leq b$  then  $a \leq E(g(X, Y)) \leq b$ . You may assume that  $X$  and  $Y$  are continuous random variables.
- Suppose  $X_1, \dots, X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$ . We know that  $(n-1)S^2/\sigma^2 \sim \chi_{n-1}^2$ . Use this fact to find  $E(S)$ .
- Let  $X_1, \dots, X_n \stackrel{iid}{\sim} Geom(p)$ . Find the distribution of  $Y = \sum X_i$ .
- Let  $X$ ,  $Y$ , and  $Z$  be three random variables with common variance  $\sigma^2$  and pairwise correlations  $\rho_{XY} = 0.3$ ,  $\rho_{XZ} = 0.5$  and  $\rho_{YZ} = 0.2$ . Find  $Cor(X + Y, Y + Z)$ .