Problem 1. Show that if \( f, g : X \to Y \) are continuous, where \( Y \) is a Hausdorff space, then the set

\[
F = \{ x \in X \mid f(x) = g(x) \}
\]

is a closed subset of \( X \).

Problem 2. Let \( X \) and \( Y \) be the two subspaces of the plane \( \mathbb{R}^2 \) defined as follows: \((a, b) \in X\) if and only if \( a \) and \( b \) are both irrational; \((a, b) \in Y\) if and only if \( a \) is irrational or \( b \) is irrational, or both. Prove that one of the spaces \( X \) or \( Y \) is connected and the other is not.

Problem 3. Suppose that \((X, d)\) is a compact metric space which has the following property: for any points \( x \) and \( y \) of \( X \), and for any \( \epsilon > 0 \), there exists a finite sequence \( \{x = x_0, x_1, x_2, \ldots, x_n = y\} \) of points of \( X \) such that \( d(x_{i-1}, x_i) < \epsilon \) for \( i \in \{1, 2, \ldots, n\} \). Prove that \( X \) is connected.