Problem 1. If $A \subset X$ and $B \subset Y$, show that

$$\text{Bd}(A \times B) = (\text{Bd}(A) \times B) \cup (A \times \text{Bd}(B))$$

Problem 2. Show that if $f, g : X \to Y$ are continuous, where $Y$ is a Hausdorff space, then the set

$$F = \{ x \in X | f(x) = g(x) \}$$

is a closed subset of $X$.

Problem 3. Show that if $f, g : X \to Y \times Z$ are continuous, then the space

$$W = \{ (x, f(x), g(x)) | x \in X \} \subset X \times Y \times Z$$

is homeomorphic to $X$.

Problem 4. Suppose that $(X, d)$ is a compact metrizable space which has the following property: for any points $x$ and $y$ of $X$, and for any $\epsilon > 0$, there exists a finite sequence $\{x = x_0, x_1, x_2, \ldots, x_n = y\}$ of points of $X$ such that $d(x_{i-1}, x_i) < \epsilon$ for $i \in \{1, 2, \ldots, n\}$. Prove that $X$ is connected.

Problem 5. Let $X$ and $Y$ be the two subspaces of the plane $\mathbb{R}^2$ defined as follows: $(a, b) \in X$ if and only if $a$ and $b$ are both irrational; $(a, b) \in Y$ if and only if $a$ is irrational or $b$ is irrational, or both. Prove that one of the spaces $X$ or $Y$ is connected and the other is not.