# Topology Masters Comp

**March 23, 2005**

1. Let $X$ be a topological space and let $A_1, A_2, \cdots$ be subsets of $X$. Prove or give a counter example ($\overline{A}$ = closure of $A$):

   (a) $\overline{A_1 \cup A_2} = \overline{A_1} \cup \overline{A_2}$

   (b) $\overline{A_1 \cap A_2} = \overline{A_1} \cap \overline{A_2}$

   (c) $\overline{\bigcup_{n=1}^{\infty} A_n} = \bigcup_{n=1}^{\infty} \overline{A}_n$

2. Let $\mathbb{R}$ be the real numbers with the usual topology. Prove or give a counterexample:

   (a) If $A$ and $B$ are compact subsets of $\mathbb{R}$ then $A + B \equiv \{a + b : a \in A,\ b \in B\}$ is a compact subset of $\mathbb{R}$

   (b) If $A$ and $B$ are connected subsets of $\mathbb{R}$ then $A + B$ is a connected subset of $\mathbb{R}$.

3. Let $X$ be a Hausdorff space and suppose that $h : X \to X$ is a homeomorphism and $g : X \to X$ is continuous. Prove that $\{(h(x), g(x)) : x \in X\}$ is a closed subset of $X \times X$ (with the product topology).