Instructions: Work at most one problem per side of the furnished paper.

(1) Suppose $A, X,$ and $Y$ are topological spaces. Give $X \times Y$ the product topology. Suppose $\pi_X : X \times Y \to X$ and $\pi_Y : X \times Y \to Y$ are given by $(x, y) \mapsto x$ and $(x, y) \mapsto y$. Prove that a function $f : A \to X \times Y$ is continuous if and only if $\pi_X \circ f$ and $\pi_Y \circ f$ are continuous.

(2) Suppose $(X, d)$ is a metric space such that the set $A := \{d(x_1, x_2) : x_i \in X, i = 1, 2\}$ is the closed interval $[0, 1]$.
   (a) Is $X$ necessarily compact?
   (b) Is $X$ necessarily connected?

(3) Suppose $\{0, 1\}$ has the discrete topology, and the countably infinite product $X = \prod_{n \in \mathbb{N}} \{0, 1\}$ has the product topology. Find the closures of the following subsets of $X$:
   (a) $A = \{(0, 0, 0, \ldots), (1, 0, 0, \ldots), (1, 1, 0, 0, \ldots), (1, 1, 1, 0, 0, \ldots), \ldots\}$.
   (b) $B = \{(x_n) : x_n = 0 \text{ for infinitely many values of } n\}$

(4) Suppose $X := \mathbb{R}^2 - (y\text{-axis})$ has the subspace topology.
   (a) Show that $X$ is locally compact.
   (b) Define $S^2$ to be the one-point compactification of $\mathbb{R}^2$. Is the one-point compactification $\overline{X}$ of $X$ homeomorphic to $S^2$?