Please choose 4 of the following 5 problems to work.

Problem 1.

a. Give the definition of a connected topological space.

b. If X is connected and $f : X \to Y$ is continuous, prove that $f(X)$ is connected.

Problem 2. Prove that the arbitrary product of Hausdorff spaces is Hausdorff.

Problem 3. Let X be $\mathbb{R}$ be with the countable complement topology, i.e. $U \subset \mathbb{R}$ is open if and only if $U = \emptyset$ or the complement $\mathbb{R} - U$ is countable.

a. Find the closure and the interior of $Y = \mathbb{R}_+ \subset X$.

b. Is $f : X \to X$ given by $f(x) = \cos(x)$ continuous?

c. Is X compact?

d. Is X Hausforff?

Explain your answers.

Problem 4. Let $f : \mathbb{R} \to \mathbb{R}$ be a continous map with the property that the image of any unbounded subset of $\mathbb{R}$ is also unbounded. Assume the standard topology on $\mathbb{R}$.

a. Show that $f^{-1}(K)$ is compact for each compact subset K of $\mathbb{R}$.

b. Show that f is a closed mapping.

Problem 5. Let $f : [0,1] \to X$ be a path that begins and ends at x_0, i.e., $[f] \in \pi_1(X, x_0)$. Let $g : [0,1] \to [0,1]$ be a homeomorphism. Prove that $[f \circ g] = [f]$ or $[f]^{-1}$.