1. Let $f$ be analytic with a zero of order $N \geq 1$ at $z_0$. Show that there exists an analytic $N$-th root of $f$ in a neighborhood of $z_0$, i.e., an analytic function $g$ such that $g(z)^N = f(z)$ in some disk about $z_0$.

2. Find a conformal map from the half-strip $D = \{z = x + iy \mid 0 < x < 1, y > 0\}$ onto the upper half-plane $H = \{z = x + iy \mid y > 0\}$. (Hint: Trigonometric functions might help.)

3. Let $f$ be an entire function with $\limsup_{z \to \infty} \frac{\log |f(z)|}{\log |z|} < \infty$. Show that $f$ is a polynomial. (Hint: First show that the assumption implies that there exist $N > 0$ and $R > 0$ such that $|f(z)| \leq |z|^N$ for $|z| \geq R$.)

4. Find
\[
\int_0^\infty \frac{dx}{x^6 + 1}
\]

5. Does there exists a function $f$ which is analytic in the unit disk and satisfies $f(1/n) = f(-1/n) = 1/n^3$ for all $n = 2, 3, \ldots$? Justify your answer.