1. Using the principal branch definition for $z^i$ determine the set of all $z \in \mathbb{C}$ for which $(z^i)^2 = (z^2)^i$.

2. Suppose $f(z)$ is analytic on $|z| < 127$, has zero as a root and $\text{Im}(f(z)) = y(4x - 1)$, $z = x + iy$.
Determine a formula for $f(z)$.

3. The function 
   
   $$f(z) = \frac{1}{z^2 \log(1 + 4z)}$$
   
   has a pole of order 3 at $z = 0$ (here $\log(z)$ is principal branch).
   
   a) Determine the first three terms a Laurent expansion for $f(z)$ convergent on an annular region centered at $z = 0$.
   
   b) Compute the integral 
   
   $$I = \int_{|z|=0.00001} f(z) \, dz$$
   
   where the orientation of the circle is counterclockwise.

4. Using an appropriate branch for $z \mapsto \sqrt{z}$, evaluate the following indefinite integral:
   
   $$I = \int_{1}^{\infty} \frac{\sqrt{x-1}}{x^2} \, dx$$

5. Let 
   
   $$f(z) = \frac{i - z}{i + z}$$
   
   and define 
   
   $$S(z) = \sum_{n=0}^{\infty} f(z)^n$$
   
   a) By using the binomial theorem and summing the series $S(z)$, find a meromorphic function $g(z)$ defined for all $z \neq 0$ that is equal to $S(z)$ wherever it converges.
   
   b) Using the fact that $f(z)$ is a linear fractional transformation, determine the set of $z$ for which $S(z)$ converges.