Program Learning Outcomes

Students demonstrate the ability to:

1) Reason with and about mathematical statements and construct and validate mathematical arguments (M 242).
2) Solve problems with and reason about functional relationships and algebraic structures (M 328).
3) Apply fundamental ideas of number theory and combinatorics in the exploration, solution, and formulation of problems (M 328).
4) Create, critique, and revise proofs in Euclidean and non-Euclidean geometries (M 329).
5) Model, analyze, and interpret situations using data analysis, statistics, and probability (M 428).
6) Develop, apply and validate mathematical models using current and emerging technologies (M 428).

Threshold

For the students completing the program in mathematics teaching, our goal is that 100% of students will be at an acceptable level or better, and 50% will be at a proficient level, for each of the learning outcomes.

Fall 2016 Assessment Process

Assessed by: Kim Nordby and Elizabeth Burroughs

For the Fall of 2016, two learning outcomes were assessed in the course M328: Higher Mathematics for Secondary Teachers. As stated in the assessment plan, by completing the math education teaching option, students were assessed on their ability to:

- Solve problems with and reason about functional relationships and algebraic structures. (Outcome 2)
- Apply fundamental ideas of number theory and combinatorics in the exploration, solution, and formulation of problems. (Outcome 3)

According the instructor’s choice, these outcomes were assessed using two clusters of representative items from the final exam for M328: Higher Mathematics for Secondary Teachers. For each cluster, a student’s responses to all items on that cluster was given a score, according the following rubric from the assessment plan.
Assessment Items
Cluster 1: Items 1, 3, and 9 on the final exam assessed students’ ability to solve problems with and reason about functional relationships and algebraic structures. The items are listed below.

1. Recall the Shaq-Jordan problem described in chapter 1. Recall that the equation \( s = \frac{30j + 1084}{41} \) gives the values of \( s \) and \( j \) for which the season averages of Shaq and Jordan would be equal, where \( s \) is the number of points Shaq scores in the final game and \( j \) is the number of points Jordan scores in the final game. Solve the equation for \( j \). What is the meaning of the slope and \( j \)-intercept of the line that is the graph of the resulting equation. (Explain thoroughly.)

3. Prove that the set of even integers form a group under the operation of addition.

9. Give an example of a Diophantine equation that has no solutions. Give a mathematical reason for why the equation has no solution.

Cluster 2: Items 5, 8, and 10 on the final exam assessed students’ ability to apply fundamental ideas of number theory and combinatorics in the exploration, solution, and formulation of problems.

5. Consider the following problem:
   At a museum, an adult ticket costs $10 and a student ticket costs $7. Is it possible for a group of adults and students to spend exactly $156 on tickets for a museum visit? Determine whether this problem has solutions or not. If it does, provide a solution; if it does not, explain why not.

8. Prove why the following statement must be true.
   If \( ab \equiv 0 \text{ Mod}(11) \), then \( a \equiv 0 \text{ Mod}(11) \) or \( b \equiv 0 \text{ Mod}(11) \).
   Does the statement hold in general, i.e. is it always true that:
   If \( ab \equiv 0 \text{ Mod}(m) \), then \( a \equiv 0 \text{ Mod}(m) \) or \( b \equiv 0 \text{ Mod}(m) \).

10. Explain in your own words why, for any integers \( a \) and \( b \), \( \text{gcf}(a,b) \times \text{lcm}(a,b) = |ab| \)

Assessment Results
There were 12 students in the course, all mathematics teaching majors.

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<th>Number (percentage) of students achieving this level</th>
<th>Unacceptable Level</th>
<th>Acceptable Level</th>
<th>Proficient Level</th>
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<tr>
<td>3</td>
<td>5</td>
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Learning Outcome 1:
25% of the students in the course demonstrated an unacceptable understanding of solving problems with and reasoning about functional relationships and algebraic structures. 75% of the students in the course demonstrated an acceptable or better understanding of solving problems with and reasoning about functional relationships and algebraic structures. 33% of the students demonstrated a proficient understanding of solving problems with and
reasoning about functional relationships and algebraic structures. It should be noted that all the students who demonstrated an unacceptable ability level in this outcome did not pass the course and will retake the course in order to complete the program.

The 3 students whose work demonstrated an unacceptable understanding in this outcome all showed a lack of understanding of group structures and properties. Additionally, their overall responses to the selected questions lacked awareness that in this context, the solution process is more important than the answer.

Learning Outcome 2:

25% of the students in the course demonstrated an unacceptable understanding of applying fundamental ideas of number theory and combinatorics in the exploration, solution, and formulation of problems. 75% of the students in the course demonstrated an acceptable or better understanding of applying fundamental ideas of number theory and combinatorics in the exploration, solution, and formulation of problems. 33% of the students demonstrated a proficient ability level in applying fundamental ideas of number theory and combinatorics in the exploration, solution, and formulation of problems. It should again be noted that all the students who demonstrated an unacceptable ability level in this outcome did not pass the course.

The 3 students whose work demonstrated an unacceptable understanding in this outcomes showed a lack of understanding of the basic aspects number theory. They did not explore questions in depth and often assumed specific examples to generalize. In general, their answers demonstrated lack of sophistication to explore if and why a certain result generalizes or does not generalize.

Recommendations

The program has moved the focus on combinatorial content from M 328 to M 242. Consequently the assessment of understanding of combinatorics should move to M 242. The syllabus for M 328 has been revised to allow more time to focus on functional and algebraic relationships and number theory. With more time devoted to exploring these concepts, students will have more support in understanding these concepts at a deeper level.

Because students who did not meet an acceptable level must retake the course in order to complete the program, our program has met the threshold that 100% of students who complete the program will be at an acceptable level or better. Of the students moving forward in the program, 4 out of 9 are proficient, which is just shy of the 50% threshold.

Spring 2017 Assessment Process
Assessed by Elizabeth Arnold and Elizabeth Burroughs

For the Spring of 2017, one learning outcome was assessed in the course M329: Geometry. As stated in the assessment plan, by completing the math education teaching option, students were assessed on their ability to:

- Create, critique, and revise proofs in Euclidean and non-Euclidean geometries.
  (Outcome 4)

According to the instructor’s choice, this outcome was assessed using a cluster of representative items from the final exam for M329: Geometry. For each cluster, a student’s responses to all items in that cluster was given a score, according the following rubric from the assessment plan.
**Assessment Items**

- Item 3 on the final exam assessed students’ ability to critique and revise a Euclidean proof: Read through the student’s proof and critique it. Provide detailed and helpful feedback directly on the proof. Now, complete your version of the proof.
- Item 4a on the final exam assessed students’ ability to create a Euclidean proof: Write a complete proof of the ASA criterion for congruent triangles theorem using a transformational proof.
- Item 6 on the final exam assessed students’ ability to create a non-Euclidean proof: In hyperbolic geometry, state and prove a property of a quadrilateral with four congruent angles and a pair of adjacent sides congruent.

**Assessment Results**

There were 9 students in the course, all of them mathematics teaching majors.

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<tr>
<th>Number (percentage) of students achieving this level</th>
<th>Unacceptable Level</th>
<th>Acceptable Level</th>
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89% of the students in the course demonstrated an *acceptable* level in this learning outcome. Further, 56% demonstrated a *proficient* level for the outcome.

Because students who did not meet an acceptable level must retake the course in order to complete the program, our program has met the threshold that 100% of students who complete the program will be at an acceptable level or better. Of the students moving forward in the program, 5 out of 8 are proficient, which meets 50% threshold.

**Recommendations**

Overall, students performed strongest in creating, critiquing, and revising proofs in Euclidean geometry. Two of the students struggled with creating a viable proof for the Euclidean task; however, these two students demonstrated the ability to complete this proof and meet this learning outcome during the course via completion of a homework assignment. Students performed weakest in working with non-Euclidean (e.g., Hyperbolic) geometry. This was to be expected, as we spent the least amount of time in the course working on Hyperbolic geometry. Instruction in the future could involve spending more time (approximately three weeks) on non-Euclidean geometry and giving students greater opportunity to complete and discuss non-Euclidean constructions, conjectures, and viable proofs. To create more time...
towards the end of the semester, it is recommended to give one midterm, rather than two, throughout the course.