Practice Final, MATH 224, Spring 2007

1. (20 pts) True or false? Correct the false statements.

(a) If two vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel, then $\mathbf{a} \times \mathbf{b} = \mathbf{0}$.

(b) The equation $3x + y - z + 2 = 0$ describes a line.

(c) The equation $x^2 + y^2 + z^2 = 1 + x$ describes a sphere.

(d) If an object moves at constant speed, then its acceleration is zero.

(e) If the acceleration of an object is zero, then it moves at constant speed.

(f) If f is differentiable and has a local maximum or minimum at $\mathbf{x}$, then $\nabla f(\mathbf{x}) = \mathbf{0}$.

(g) If f is differentiable and $\nabla f(\mathbf{x}) = \mathbf{0}$, then f has a local maximum or minimum at $\mathbf{x}$.

(h) If D is the upper half of the unit disk and f is a continuous function, then $\iint_D f(x, y)\, dx\, dy = \int_0^1 \int_0^\pi f(r\cos\theta, r\sin\theta)\, d\theta\, dr$.

(i) If a vector field $\mathbf{F} = \langle P, Q \rangle$ is conservative, then $\dfrac{\partial P}{\partial x} = \dfrac{\partial Q}{\partial y}$

(j) If $f(x, y)$ is a continously differentiable function, then $\int_C \nabla f \cdot d\mathbf{r} = 0$ for any simple closed curve C.

2. (8 pts) A constant force $\mathbf{F} = \langle 1, 2, -3 \rangle$ moves an object along a line segment from $(2, 0, 1)$ to $(1, 3, 2)$. Find the work done if the distance is measured in meters and the force in Newtons.

3. (8 pts) A plane contains the points $(2, 1, 0)$, $(-1, 1, 1)$, and $(0, 0, 2)$. Find a vector which is perpendicular to the plane. Where does the plane intersect the x-axis?

4. (8 pts) Identify and sketch the surface $x = y^2 + z^2 - 4z$.

5. (8 pts) Find the linearization of $f(x, y) = \sqrt{x^2 + 2y^2}$ at the point $(1, 2)$ and use it to estimate $f(1.1, 1.9)$.

6. (8 pts) Let $f(x, y, z) = xy + z^2$. Find the directional derivative of f at the point $(3, 0, -2)$ in the direction of $\mathbf{v} = \langle -1, 2, 2 \rangle$. What are the directions of maximal and minimal directional derivatives at $(3, 0, -2)$?

7. (10 pts) Let $f(x, y) = x^2y + y^3 - y$. Find all critical points of f and determine whether they are maxima, minima, or saddle points.

8. (10 pts) Evaluate $\int_0^4 \int_{\sqrt{x}}^2 e^{y^3}\, dy\, dx$ by reversing the order of integration.

9. (10 pts) Consider a lamina that occupies the region $4 \leq x^2 + y^2 \leq 9$, with mass density $\rho(x, y) = (x^2 + y^2)^{-3/2}$. Find the moments of inertia of the lamina about the coordinate axes.

10. (10 pts) Use Green's Theorem to evaluate $\int_C (y + \sin\sqrt{x})dx + (3x - \ln(1 + y^2))dy$, where C is the circle $(x-3)^2+(y-1)^2 = 4$, parameterized in counterclockwise direction. (Hint: You may use the fact that the area of a circle of radius r is πr^2.)