First Test, MATH 224, Fall 2006 (slightly modified)

These might be useful: $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, $\sin\frac{\pi}{6} = \frac{1}{2}$, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $(1+2t)^2 = 1+4t+4t^2$.

1. (5 pts) Which of the points $(0,1,2)$, $(3,4,1)$, $(-2,0,-3)$, $(1,-1,2)$ is closest to the xy-plane? Which point is in the xz-plane?

2. (5 pts) Which of the following expressions are meaningful? Which are meaningless? For the meaningful ones, is the result a vector or a scalar?

- $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$
- $(\mathbf{a} \cdot \mathbf{b}) + \mathbf{c}$
- $(|\mathbf{a}|\mathbf{b}) \cdot \mathbf{c}$
- $|\mathbf{a} + \mathbf{b}| \times \mathbf{c}$
- $(\mathbf{a} - \mathbf{b}) \times ((\mathbf{a} \cdot \mathbf{b})\mathbf{b})$

3. (10 pts) Which of the following statements are true, which are false?

- $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
- $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$
- $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = \mathbf{0}$
- $\frac{d}{dt}(\mathbf{u}(t) \cdot \mathbf{v}(t)) = \mathbf{u}'(t) \cdot \mathbf{v}'(t)$
- $\frac{d}{dt}(\mathbf{u}(t) \times \mathbf{u}(t)) = \mathbf{0}$

4. (10 pts) What is the dot product of two vectors with lengths 3 and 4, if the angle between them is 30°? If we know in addition that both vectors lie in the xy-plane, what can be said about their cross product?

5. (10 pts) Write down an equation for the plane which contains the points $(1,2,3)$, $(2,3,4)$, and $(3,4,6)$. Which of the points $(0,1,2)$ and $(0,2,1)$ lies in this plane?

6. (10 pts) What can you say about a planar curve if the curvature satisfies $\kappa = 0$ everywhere along the curve? What if $\kappa = 2$ instead?

7. (20 pts) A river flowing east is 10m wide, and the water speed in the river is given by the function $f(x) = \frac{1}{5}x(10-x)$ (in m/s), where x is the distance from the north bank in meters. A boat proceeds with a constant speed of 2 m/s from a point A on the north bank, heading straight south. How far down the river will the boat arrive on the south bank?

8. (15 pts) A wagon is pulled a distance of 50 m by a constant force of 20 N. The handle of the wagon is held at an angle of 45°. How much work is done?

9. (15 pts) Find the length of the curve $\mathbf{r}(t) = \langle 3t+1, 4t^{3/2}-1, 3t^2 \rangle$, $0 \le t \le 1$.