1. Let \( f \) be a quadratic polynomial with two distinct roots \( a \neq b \). Show that the associated Newton’s method \( N_f(z) = z - \frac{f(z)}{f'(z)} \) is conformally conjugate to the Newton’s method for \( f_0(z) = z^2 - 1 \). Conclude that Newton’s method with initial value \( z_0 \in \mathbb{C} \) converges to \( a \) iff \( |z_0 - a| < |z_0 - b| \), and that it converges to \( b \) iff \( |z_0 - b| < |z_0 - a| \). (Hint: Use the results about Newton’s method for \( f_0 \) from class.)

2. Let \( T \) be a Möbius transformation which is not the identity, and assume that \( z_0 \in \mathbb{C} \) is a fixed point of \( T \) with \( T'(z_0) = 1 \). Show that \( T^n(z) \to z_0 \) as \( n \to \infty \), for all \( z \in \hat{\mathbb{C}} \). (Hint: Derivatives of fixed points are invariant under analytic conjugation, and we classified the dynamical behavior of Möbius transformations in class.)

3. Let \( f \) be a quadratic polynomial. Show that \( f \) is conformally conjugate to a unique quadratic polynomial of the form \( f_c(z) = z^2 + c \).

4. Let \( |c| < 1/4 \), and let \( f_c(z) = z^2 + c \) with Julia set \( J_c \). Show that

(a) \( |f_c(z)| > |z| \) for \( |z| > \frac{1}{2} + \sqrt{\frac{1}{4} + |c|} \), and

(b) \( |f_c(z)| \leq |z| \) for \( |z| = \frac{1}{2} + \sqrt{\frac{1}{4} - |c|} \).

Conclude that for every \( \epsilon > 0 \) there exists \( \delta > 0 \) such that \( J_c \) is contained in the annulus \( \{ z \in \mathbb{C} : 1 - \epsilon < |z| < 1 + \epsilon \} \) whenever \( |c| < \delta \). (I.e., for small \( c \), the Julia set \( J_c \) is close to the unit circle.)