Math 333 Makeup of Exam 1 (take home)

Name:

If you use a theorem indicate what it is. Show all work (unless instructed otherwise).
Good Luck!

0. Circle **True** or **False** without explanation:

(**T** or **F**) Every vector space with real scalars V contains infinitely many vectors.

(**T** or **F**) Any set of nine vectors spanning $M_{3\times 3}$ is independent.

(**T** or **F**) If $T \circ S$ is onto then S is onto.

(**T** or **F**) For finite dimensional vector spaces, $\dim(U \times W) = \dim(U) \cdot \dim(W)$.

1. Show that $W := \left\{ \begin{bmatrix} a & 0 \\ b & a \end{bmatrix} : a, b \in \mathbf{R} \text{ with } a \geq 0 \right\}$ is not a subspace of $M_{3\times 3}$.

2. Given a basis of $S_{2\times 2}$, $\mathcal{B} = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$, write out explicitly the $A \in S_{2\times 2}$ for which the coordinate vector is $[A]_{\mathcal{B}} = [1, 2, 3]$. (Show your method.)

$A =$

3. Show that $T : \mathbf{R} \to \mathbf{R}$ given by $T(x) = x + 1$ is not a linear transformation.

4. Prove (from the axioms) that, in any vector space V, if $v \in V$ then $0 \cdot v = \mathbf{0}$.

5. a) Find a basis for $\text{span}(1 - 2x, 2x - x^2, 1 - x^2, 1 + x^2)$ in $\mathcal{P}_2$.

b) Is the span equal to $\mathcal{P}_2$? Justify.

6. Consider $T : \mathcal{P}_2 \to \mathcal{P}_2$ given by $(Tf)(x) := f(2x - 1)$.

a) Find the matrix of T with respect to the standard basis.

b) Find the formula for the inverse T^{-1}, that is complete $(T^{-1}g)(y) = \dots$. (Do not use part a) for this.)

7. Let $T : V \to W$ be a linear transformation.

a) Carefully show that range(T) is a subspace of W.

b) Outline the argument showing the famous formula

$$\dim(\ker(T)) + \dim(\text{range}(T)) = \dim(V).$$