4.8 Alternate Analysis as a One-way ANOVA

- Suppose we have data from a two-factor factorial design. The following method can be used to perform a multiple comparison test to compare treatment means as well as Levene’s Test to check the homogeneity of variance assumption.

- The main idea is to create a single factor having \( a \times b \) levels and analyze the data as if you had a one-way ANOVA with \( a \times b \) treatments.

Multiple Comparison Procedures

Suppose the researcher is interested in comparing the cell means from a two-factor factorial design. The following method can be used to perform a multiple comparison procedure (MCP):

1. Create a single factor having \( a \times b \) levels. For the \( 2 \times 2 \) design example, create a single factor having four levels from the two levels of time (\( T = 12, 18 \)) and the two levels of medium (\( M = 1, 2 \)). In the SAS code, I called these levels 12_1, 12_2, 18_1, and 18_2.

2. Run Bonferroni’s MCP (or any other MCP) on this single factor. For the \( 2 \times 2 \) design, we reject all \( H_0: \mu_{ij} = \mu_{ij'} \) except for \( \mu_{12_2} = \mu_{12_1} \).

Levene’s Test of the HOV Assumption

- In a two-way ANOVA, the HOV assumption implies that all \( a \times b \) variances are equal. That is, we assume \( \sigma^2_{11} = \sigma^2_{12} = \ldots = \sigma^2_{ab} \) where \( \sigma^2_{ij} \) is the variance associated with the errors for treatment combination \( (i,j) \) based on the two design factors.

- Suppose the researcher is interested in testing the HOV assumption that all of the \( a \times b \) variances are equal in a two-factor factorial design.

- The following method can be used to perform Levene’s HOV Test:

  1. Create a single factor having \( a \times b \) treatment levels. For the \( 2 \times 2 \) design example, create a single factor having four levels from the two levels of time (12,18) and the two levels of medium (1,2). In the SAS code, I called these levels 12_1, 12_2, 18_1, and 18_2.

  2. Run Levene’s HOV Test on this single factor. For the \( 2 \times 2 \) design example, we fail to reject the HOV assumption (\( p \)-value = .1793).

**Dependent Variable: growth**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>3</td>
<td>691.4583333</td>
<td>230.4861111</td>
<td>45.12</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Error</td>
<td>20</td>
<td>102.1666667</td>
<td>5.1083333</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>23</td>
<td>793.6250000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R-Square</th>
<th>Coeff Var</th>
<th>Root MSE</th>
<th>growth Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.871266</td>
<td>7.629240</td>
<td>2.260162</td>
<td>29.62500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>time_med</td>
<td>3</td>
<td>691.4583333</td>
<td>230.4861111</td>
<td>45.12</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>
Levene's Test for Homogeneity of growth Variance
ANOVA of Absolute Deviations from Group Means

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>time_med</td>
<td>3</td>
<td>6.3472</td>
<td>2.1157</td>
<td>1.80</td>
<td>0.1793</td>
</tr>
<tr>
<td>Error</td>
<td>20</td>
<td>23.4815</td>
<td>1.1741</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Alpha 0.05

Error Degrees of Freedom 20

Error Mean Square 5.108333

Critical Value of t 2.92712

Minimum Significant Difference 3.8196

Means with the same letter are not significantly different.

<table>
<thead>
<tr>
<th>Bon Grouping</th>
<th>Mean</th>
<th>N</th>
<th>time_med</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>37.167</td>
<td>6</td>
<td>18_1</td>
</tr>
<tr>
<td>B</td>
<td>32.000</td>
<td>6</td>
<td>18_2</td>
</tr>
<tr>
<td>C</td>
<td>26.000</td>
<td>6</td>
<td>12_2</td>
</tr>
<tr>
<td>C</td>
<td>23.333</td>
<td>6</td>
<td>12_1</td>
</tr>
</tbody>
</table>
DATA in;
  DO time_med = '12_1', '12_2', '18_1', '18_2';
  DO rep = 1 to 6;
    INPUT growth @@; OUTPUT;
  END; END;
CARDS;
21 23 20 22 28 26 25 24 29 26 25 27
37 38 35 39 38 36 31 29 30 34 33 35;
PROC GLM DATA=in;
  CLASS time_med;
  MODEL growth = time_med / SS3;
  MEANS time_med / BON HOVTEST=LEVENE(TYPE=ABS);
TITLE 'ALTERNATE ANOVA AND HOV TEST';
RUN;

4.9 Other Multiple Comparison Procedures

- You can also perform a MCP using the LSMEANS statement in Proc GLM in SAS. E.g., to perform a Bonferroni MCP:
  1. Include a LSMEANS A*B statement with option / ADJUST=BON for factors A and B.
  2. Reject $H_0: \mu_{ij} = \mu_{i'j'}$ if the adjusted $p$-value in the matrix of $p$-values is $\leq \alpha$. This is equivalent to taking the $p$-value $\leq \alpha^*$ where $\alpha^* = \alpha/C$ and $C$ is the number of comparisons made. In essence, we are just multiplying the individual test $p$-values by the number of comparisons, and then checking if the adjusted $p$-value is $< \alpha$.

- In SAS, it is possible to perform a test of the equality of cell means (i) across the levels of factor A for a specified level $j$ of factor B and (ii) across the levels of factor B for a specified level $i$ of factor A. These are called tests of effects slices.

- Tests of effects slices can be performed using the LSMEANS statement in Proc GLM in SAS.
  1. Include a statement of the form LSMEANS A*B / SLICE = A ADJUST = BON for a MCP of cell means across the levels of factor B for each level of factor A. The hypotheses tested for level $i$ of factor A are
     
     $H_0: \mu_{ij} = \mu_{i'j'}$
     $H_1: \mu_{ij} \neq \mu_{i'j'}$
  2. Include a statement of the form LSMEANS A*B / SLICE = B ADJUST = BON for a MCP of cell means across the levels of factor A for each level of factor B. The hypotheses tested for level $b$ of factor B are
     
     $H_0: \mu_{ij} = \mu_{ij'}$
     $H_1: \mu_{ij} \neq \mu_{ij'}$

- For a different MCP, just change BON to TUKEY, SIDAK, or SCHEFFE. If you do not include the ADJUST option, you will get Fisher’s LSD test by default.
4.10 ANOVA for a 2 × 3 Factorial Design Example

- An experiment was run to investigate how the type of glass and the type of phosphorescent coating affects the brightness of a light bulb. The response variable is the current (in microamps) to obtain a specified brightness. The data are

<table>
<thead>
<tr>
<th>Phosphor Type</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Glass Type</td>
<td>278</td>
<td>291</td>
<td>285</td>
</tr>
<tr>
<td>2 Glass Type</td>
<td>229</td>
<td>235</td>
<td>241</td>
</tr>
</tbody>
</table>

- Look at the difference in glass means across the levels of phosphor:

\[
\bar{y}_{2,A} - \bar{y}_{1,A} = -49.7 \quad \bar{y}_{2,B} - \bar{y}_{1,B} = -49.3 \quad \bar{y}_{2,C} - \bar{y}_{1,C} = -52.3
\]

The variability in these three differences (\(MS_{\text{glass} \times \text{phosphor}} = 4.05\)) is small relative to the \(MS_E = 44.2\), so we fail to reject the null hypothesis that the interaction effects are equal.

- The Glass*Phosphor interaction is not significant (\(p\)-value = .9130). This is also obvious from the strong parallelism in the interaction plots.

- The Bonferroni MCT results are summarized below:

<table>
<thead>
<tr>
<th>Glass/Phosphor</th>
<th>2 C</th>
<th>2 A</th>
<th>2 B</th>
<th>1 C</th>
<th>1 A</th>
<th>1 B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>229.3</td>
<td>235.0</td>
<td>249.7</td>
<td>281.7</td>
<td>284.7</td>
<td>299.0</td>
</tr>
</tbody>
</table>

SAS Code and Output

DM 'LOG; CLEAR; OUT; CLEAR;';
ODS GRAPHICS ON;
ODS PRINTER PDF file='C:\COURSES\ST541\TWOWAY2.PDF';
OPTIONS NODATE NONUMBER;
***************************************;
*** 2-FACTOR FACTORIAL (2x3) DESIGN ***;
***************************************;
DATA in; INPUT glass phosphor $ light @@; CARDS;
  1 A 278 1 A 291 1 A 285 1 B 297 1 B 304 1 B 296
  1 C 273 1 C 284 1 C 288 2 A 229 2 A 235 2 A 241
  2 B 259 2 B 249 2 B 241 2 C 225 2 C 228 2 C 235
; PROC GLM DATA=in ;
  CLASS glass phosphor;
  MODEL light = glass|phosphor / SS3 SOLUTION;
  MEANS glass|phosphor / BON;
  LSMEANS glass*phosphor / SLICE=glass SLICE=phosphor ADJUST=BON;;
  ESTIMATE 'mu' intercept 1;
  ESTIMATE 'glass=1' glass 1 -1 / divisor = 2 ;
  ESTIMATE 'glass=2' glass -1 1 / divisor = 2 ;
  ESTIMATE 'phosphor=A' phosphor 2 -1 -1 / divisor = 3 ;
  ESTIMATE 'phosphor=B' phosphor -1 2 -1 / divisor = 3 ;
  ESTIMATE 'phosphor=C' phosphor -1 -1 2 / divisor = 3 ;
ESTIMATE 'glass=1 phos=A' glass*phosphor 2 -1 -1 -2 1 1 / divisor = 6 ;
ESTIMATE 'glass=1 phos=B' glass*phosphor -1 2 -1 1 -2 1 / divisor = 6 ;
ESTIMATE 'glass=1 phos=C' glass*phosphor -1 -1 2 1 1 -2 / divisor = 6 ;
ESTIMATE 'glass=2 phos=A' glass*phosphor -2 1 1 2 -1 -1 / divisor = 6 ;
ESTIMATE 'glass=2 phos=B' glass*phosphor 1 -2 1 -1 2 -1 / divisor = 6 ;
ESTIMATE 'glass=2 phos=C' glass*phosphor 1 1 -2 -1 1 2 / divisor = 6 ;

TITLE '(2 x 3) TWO FACTOR ANALYSIS OF VARIANCE';
RUN;

(2 x 3) TWO FACTOR ANALYSIS OF VARIANCE

The GLM Procedure

Variable: light

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>5</td>
<td>12626.44444</td>
<td>2525.28889</td>
<td>57.10</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Error</td>
<td>12</td>
<td>530.66667</td>
<td>44.22222</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>17</td>
<td>13157.11111</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

R-Square  Coeff Var  Root MSE  light Mean
0.959667  2.526375  6.649979  263.2222

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>glass</td>
<td>1</td>
<td>11450.88889</td>
<td>11450.88889</td>
<td>258.94</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>phosphor</td>
<td>2</td>
<td>1167.44444</td>
<td>583.72222</td>
<td>13.20</td>
<td>0.0009</td>
</tr>
<tr>
<td>glass*phosphor</td>
<td>2</td>
<td>8.11111</td>
<td>4.05556</td>
<td>0.09</td>
<td>0.9130</td>
</tr>
</tbody>
</table>

| Parameter     | Estimate   | Standard Error | t Value | Pr > |t| |
|---------------|------------|----------------|---------|-------|
| Intercept     | 229.333333 | B 3.83936723    | 59.73   | <.0001|
| glass 1       | 52.333333  | B 5.42968521    | 9.64    | <.0001|
| glass 2       | 0.0000000  | B               | .       | .     |
| phosphor A    | 5.6666667  | B 5.42968521    | 1.04    | 0.3172|
| phosphor B    | 20.3333333 | B 5.42968521    | 3.74    | 0.0028|
| phosphor C    | 0.0000000  | B               | .       | .     |
| glass*phosphor 1 A | -2.6666667 | B 7.67873446   | -0.35   | 0.7344|
| glass*phosphor 1 B | -3.0000000 | B 7.67873446   | -0.39   | 0.7029|
| glass*phosphor 1 C | 0.0000000  | B               | .       | .     |
| glass*phosphor 2 A | 0.0000000  | B               | .       | .     |
| glass*phosphor 2 B | 0.0000000  | B               | .       | .     |
| glass*phosphor 2 C | 0.0000000  | B               | .       | .     |
| Parameter | Estimate | Standard Error | t Value | Pr > |t| |
|-----------|----------|----------------|---------|------|---|
| mu        | 263.222222 | 1.56741511     | 167.93  | <.0001 | |
| glass=1   | 25.222222  | 1.56741511     | 16.09   | <.0001 | |
| glass=2   | -25.222222 | 1.56741511     | -16.09  | <.0001 | |
| phosphor=A| -3.388889  | 2.21665970     | -1.53   | 0.1522 | |
| phosphor=B| 11.111111  | 2.21665970     | 5.01    | 0.0003 | |
| phosphor=C| -7.722222  | 2.21665970     | -3.48   | 0.0045 | |
| glass=1 phos=A | -0.388889 | 2.21665970 | -0.18   | 0.8637 | |
| glass=1 phos=B | -0.555556 | 2.21665970 | -0.25   | 0.8063 | |
| glass=1 phos=C | 0.944444 | 2.21665970 | 0.43    | 0.6776 | |
| glass=2 phos=A | 0.388889 | 2.21665970 | 0.18    | 0.8637 | |
| glass=2 phos=B | 0.555556 | 2.21665970 | 0.25    | 0.8063 | |
| glass=2 phos=C | -0.944444 | 2.21665970 | -0.43   | 0.6776 | |
### ANOVA Results

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha</td>
<td>0.05</td>
</tr>
<tr>
<td>Error Degrees of Freedom</td>
<td>12</td>
</tr>
<tr>
<td>Error Mean Square</td>
<td>44.22222</td>
</tr>
<tr>
<td>Critical Value of t</td>
<td>2.17881</td>
</tr>
<tr>
<td>Minimum Significant Difference</td>
<td>6.8302</td>
</tr>
</tbody>
</table>

### Means with the Same Letter Are Not Significantly Different

<table>
<thead>
<tr>
<th>Bon Grouping</th>
<th>Mean</th>
<th>N</th>
<th>phosphor</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>288.444</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>238.000</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>259.833</td>
<td>6</td>
<td>A</td>
</tr>
<tr>
<td>B</td>
<td>255.500</td>
<td>6</td>
<td>C</td>
</tr>
</tbody>
</table>

### Distribution of Light

#### Distribution Table

<table>
<thead>
<tr>
<th>Level of glass</th>
<th>Level of phosphor</th>
<th>N</th>
<th>light</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>1</td>
<td>A</td>
<td>3</td>
<td>284.666667</td>
</tr>
<tr>
<td>1</td>
<td>B</td>
<td>3</td>
<td>299.000000</td>
</tr>
<tr>
<td>1</td>
<td>C</td>
<td>3</td>
<td>281.666667</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>3</td>
<td>235.000000</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>3</td>
<td>249.666667</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>3</td>
<td>229.333333</td>
</tr>
</tbody>
</table>
### The GLM Procedure

#### Least Squares Means

**Adjustment for Multiple Comparisons: Bonferroni**

<table>
<thead>
<tr>
<th>glass</th>
<th>phosphor</th>
<th>light LSMEAN</th>
<th>LSMEAN Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>284.666667</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>B</td>
<td>299.000000</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>C</td>
<td>281.666667</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>235.000000</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>249.666667</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>229.333333</td>
<td>6</td>
</tr>
</tbody>
</table>

**Least Squares Means for effect glass*phosphor**

Pr > |t| for H0: LSMean(i)=LSMean(j)

<table>
<thead>
<tr>
<th>i/j</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3237</td>
<td>1.0000</td>
<td>&lt;.0001</td>
<td>0.0005</td>
<td>&lt;.0001</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.3237</td>
<td>0.1161</td>
<td>&lt;.0001</td>
<td>&lt;.0001</td>
<td>&lt;.0001</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.0000</td>
<td>0.1161</td>
<td>&lt;.0001</td>
<td>0.0011</td>
<td>&lt;.0001</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>&lt;.0001</td>
<td>&lt;.0001</td>
<td>&lt;.0001</td>
<td>0.2890</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.0005</td>
<td>&lt;.0001</td>
<td>0.0011</td>
<td>0.2890</td>
<td>0.0420</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>&lt;.0001</td>
<td>&lt;.0001</td>
<td>&lt;.0001</td>
<td>1.0000</td>
<td>0.0420</td>
<td></td>
</tr>
</tbody>
</table>

#### The GLM Procedure

#### Least Squares Means

**glass*phosphor Effect Sliced by glass for light**

<table>
<thead>
<tr>
<th>glass</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>514.8888889</td>
<td>257.4444444</td>
<td>5.82</td>
<td>0.0171</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>660.666667</td>
<td>330.333333</td>
<td>7.47</td>
<td>0.0078</td>
</tr>
</tbody>
</table>

#### The GLM Procedure

#### Least Squares Means

**glass*phosphor Effect Sliced by phosphor for light**

<table>
<thead>
<tr>
<th>phosphor</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>3700.166667</td>
<td>3700.166667</td>
<td>83.67</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>3650.666667</td>
<td>3650.666667</td>
<td>82.55</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>4108.166667</td>
<td>4108.166667</td>
<td>92.90</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>
4.11 Example: Sample size determination and power estimation

Determine $N$ given a nominal power level (Case 1) and determine power given $N$ (Case 2) for a specified pattern of means or effects

- Suppose there are 6 treatments resulting from a $2 \times 3$ factorial design having $n$ replicates, and based on a prior study we have estimates of the treatment means: $\mu_{11} = 35.1$, $\mu_{12} = 33.7$, $\mu_{13} = 30.2$, $\mu_{21} = 23.0$, $\mu_{22} = 25.9$, $\mu_{23} = 30.4$.

- Our prior estimate of $\sigma$ is 1.4, and the significance level is set to $\alpha = .05$ for tests.

- For Case 1, determine the total sample size $N = 6n$ setting the power for the ANOVA $F$-tests for main effects and the interaction at levels $1 - \beta = .50$, .80, .90, and .95. Also, determine $N$ for several contrasts.

- For Case 2, determine the power $1 - \beta$ for the ANOVA $F$-tests for the main effects and the interaction when the total sample size $N = 18$, 24, 30, and 36. Also, determine power for the tests of several contrasts.

SAS code for Case 1: Determine $N$ for a nominal power level

data twoway;
  input levelA $ levelB $ meanest;
datalines;
A1 B1 35.1 A1 B2 33.7 A1 B3 30.2 A2 B1 23.0 A2 B2 25.9 A2 B3 30.4
;
proc glmpower data=twoway;
  class levelA levelB;
  model meanest = levelA|levelB;
  contrast 'A1-A2' levelA 1 -1 ;
  contrast 'B1-B2' levelB 1 -1 0;
  contrast 'B1-B3' levelB 1 0 -1;
  contrast 'B2-B3' levelB 0 1 -1;
  contrast 'A11-A12' levelA 1 -1 0 levelA*levelB 1 -1 0 0 0 0;
  contrast 'A12-A23' levelA 1 -1 levelB 0 1 -1 levelA*levelB 0 1 0 0 0 -1;
  power
    stddev = 1.4
    alpha = 0.05
    ntotal = .
    power = .5 .8 .9 .95 ;
title 'Determining design size for given power and mean estimates';
title2 'for a twoway (2 x 3) ANOVA';
proc glmpower data=twoway;
  class levelA levelB;
  model meanest = levelA|levelB;
  contrast 'A1-A2' levelA 1 -1 ;
  contrast 'B1-B2' levelB 1 -1 0;
  contrast 'B1-B3' levelB 1 0 -1;
  contrast 'B2-B3' levelB 0 1 -1;
  contrast 'A11-A12' levelA 1 -1 0 levelA*levelB 1 -1 0 0 0 0;
  contrast 'A12-A23' levelA 1 -1 levelB 0 1 -1 levelA*levelB 0 1 0 0 0 -1;
  power
    stddev = 1.4
    alpha = 0.05
    ntotal = 18 24 30 36
    power = . ;
title 'Determining power for a given design size and mean estimates';
title2 'for a twoway (2 x 3) ANOVA';
run;
SAS output for Case 1: Determine $N$ for a nominal power level

The GLMPOWER Procedure

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SAS output for Case 2: Determine power for a given $N$

The GLMPOWER Procedure

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