4.15 Three Factor Factorial Designs

- The **complete interaction model** for a three-factor completely randomized design is:

$$y_{ijkl} \;=\; \tag{35}$$

  - μ is the baseline mean,
  - τ_i, β_j, and γ_k are the main factor effects for A, B, and C, respectively.
  - $(\tau\beta)_{ij}$, $(\tau\gamma)_{ik}$ and $(\beta\gamma)_{jk}$ are the two-factor interaction effects for interactions AB, AC, and BC, respectively.
  - $(\tau\beta\gamma)_{ijk}$ are the three-factor interaction effects for the ABC interaction.
  - e_{ijkl} is the random error of the k^{th} observation from the $(i,j,k)^{th}$ treatment.
    We assume $e_{ijkl} \sim IID\, N(0,\sigma^2)$. For now, we will also assume all effects are fixed.

4.15.1 Partitioning the total sum of squares

- $SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}(y_{ijkl} - \overline{y}_{....})^2$
- Note that we can rewrite y_{ijkl} as

$$y_{ijkl} \;=\; \widehat{\mu} + \widehat{\tau}_i + \widehat{\beta}_j + \widehat{\gamma}_k + \widehat{(\tau\beta)}_{ij} + \widehat{(\tau\gamma)}_{ik} + \widehat{(\beta\gamma)}_{jk} + \widehat{(\tau\beta\gamma)}_{ijk} + e_{ijkl} \qquad \text{where}$$

$$\widehat{\mu} = \overline{y}_{....} \qquad \widehat{\tau}_i = \overline{y}_{i...} - \overline{y}_{....} \qquad \widehat{\beta}_j = \overline{y}_{.j..} - \overline{y}_{....} \qquad \widehat{\gamma}_k = \overline{y}_{..k.} - \overline{y}_{....}$$

$$\widehat{(\tau\beta)}_{ij} = \overline{y}_{ij..} - \overline{y}_{i...} - \overline{y}_{.j..} + \overline{y}_{....} \qquad \widehat{(\tau\gamma)}_{ik} = \overline{y}_{i\cdot k\cdot} - \overline{y}_{i...} - \overline{y}_{..k.} + \overline{y}_{....} \qquad \widehat{(\beta\gamma)}_{jk} = \overline{y}_{\cdot jk\cdot} - \overline{y}_{.j..} - \overline{y}_{..k.} + \overline{y}_{....}$$

$$\widehat{(\tau\beta\gamma)}_{ijk} = \overline{y}_{ijk\cdot} - \overline{y}_{ij..} - \overline{y}_{i\cdot k\cdot} - \overline{y}_{\cdot jk\cdot} + \overline{y}_{i...} + \overline{y}_{.j..} + \overline{y}_{..k.} - \overline{y}_{....} \qquad e_{ijkl} = y_{ijkl} - \overline{y}_{ijk\cdot}$$

- Substitution of these estimates into SS_T yields

$$\begin{aligned}
SS_T &= \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}(y_{ijkl} - \overline{y}_{....})^2 \\
&= \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}\left(\widehat{\tau}_i + \widehat{\beta}_j + \widehat{\gamma}_k + \widehat{(\tau\beta)}_{ij} + \widehat{(\tau\gamma)}_{ik} + \widehat{(\beta\gamma)}_{jk} + \widehat{(\tau\beta\gamma)}_{ijk} + e_{ijkl}\right)^2 \\
&= \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}\widehat{\tau}_i^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}\widehat{\beta}_j^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}\widehat{\gamma}_k \\
&+ \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}\widehat{(\tau\beta)}_{ij}^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}\widehat{(\tau\gamma)}_{ik}^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}\widehat{(\beta\gamma)}_{jk}^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}\widehat{(\tau\beta\gamma)}_{ijk}^2 \\
&+ \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}e_{ijkl}^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}(\text{all cross-products of parameter estimates})
\end{aligned}$$

Through tedious algebra, it can be shown $\sum\sum\sum\sum$ cross-products of parameter estimates = 0. Simplification of the multiple summations produces:

$$\begin{aligned}
SS_T &= bcn\sum_{i=1}^{a}\widehat{\tau}_i^2 + acn\sum_{j=1}^{b}\widehat{\beta}_j^2 + abn\sum_{k=1}^{c}\widehat{\gamma}_k + cn\sum_{i=1}^{a}\sum_{j=1}^{b}\widehat{(\tau\beta)}_{ij}^2 + bn\sum_{i=1}^{a}\sum_{k=1}^{c}\widehat{(\tau\gamma)}_{ik}^2 + an\sum_{j=1}^{b}\sum_{k=1}^{c}\widehat{(\beta\gamma)}_{jk}^2 \\
&\quad + n\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\widehat{(\tau\beta\gamma)}_{ijk}^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n}e_{ijkl}^2 \\
&= SS_A + SS_B + SS_C + SS_{AB} + SS_{AC} + SS_{BC} + SS_{ABC} + SS_E
\end{aligned}$$

PARTITIONING THREE FACTOR FACTORIAL DESIGN SUM OF SQUARES

MODEL: $y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\tau\gamma)_{ik} + (\beta\gamma)_{jk} + (\tau\beta\gamma)_{ijk} + \varepsilon_{ijkl}$

$$\left\{ \begin{array}{llll} i = 1, \ldots, a & \text{FACTOR A} & k = 1, \ldots, c & \text{FACTOR C} \\ j = 1, \ldots, b & \text{FACTOR B} & l = 1, \ldots, n & \text{REPLICATION} \end{array} \right\}$$

$$SS_T = \sum_i \sum_j \sum_k \sum_l (y_{ijkl} - \bar{y}_{....})^2$$

$$= \sum_i \sum_j \sum_k \sum_l \Big[(\bar{y}_{i...} - \bar{y}_{....}) + (\bar{y}_{.j..} - \bar{y}_{....}) + (\bar{y}_{..k.} + \bar{y}_{....}) \qquad \Big\} \text{ MAIN EFFECTS}$$

$$+ (\bar{y}_{ij..} - \bar{y}_{i...} - \bar{y}_{.j..} + \bar{y}_{....}) + (\bar{y}_{i.k.} - \bar{y}_{i...} - \bar{y}_{..k.} + \bar{y}_{....})$$

$$+ (\bar{y}_{.jk.} - \bar{y}_{.j..} - \bar{y}_{..k.} + \bar{y}_{....}) \qquad \Big\} \text{ TWO FACTOR INTERACTIONS}$$

$$+ (\bar{y}_{ijk.} - \bar{y}_{ij..} - \bar{y}_{i.k.} - \bar{y}_{.jk.} + \bar{y}_{i...} + \bar{y}_{.j..} + \bar{y}_{..k.} - \bar{y}_{....}) \qquad \Big\} \text{ THREE FACTOR INTERACTION}$$

$$+ (y_{ijkl} - \bar{y}_{ijk.}) \Big]^2 \qquad \Big\} \text{ RESIDUAL}$$

FOR BREVITY, REWRITE EACH COMPONENT:

$$SS_T = \sum_i \sum_j \sum_k \sum_l \left[I_i + J_j + K_k + (IJ)_{ij} + (IK)_{ik} + (JK)_{jk} + (IJK)_{ijk} + E_{ijkl} \right]^2$$

$$= \sum_i \sum_j \sum_k \sum_l I_i^2 + \sum_i \sum_j \sum_k \sum_l J_j^2 + \sum_i \sum_j \sum_k \sum_l K_k^2$$

$$+ \sum_i \sum_j \sum_k \sum_l (IJ)_{ij}^2 + \sum_i \sum_j \sum_k \sum_l (IK)_{ik}^2 + \sum_i \sum_j \sum_k \sum_l (JK)_{jk}^2$$

$$+ \sum_i \sum_j \sum_k \sum_l (IJK)_{ijk}^2 + \sum_i \sum_j \sum_k \sum_l E_{ijkl}^2 + 2 \sum_i \sum_j \sum_k \sum_l (\text{CROSS-PRODUCTS}) \rightarrow 0$$

$$= bcn \sum_i I_i^2 + acn \sum_j J_j^2 + abn \sum_k K_k^2$$

$$+ cn \sum_i \sum_j (IJ)_{ij}^2 + bn \sum_i \sum_k (IK)_{ik}^2 + an \sum_j \sum_k (JK)_{jk}^2$$

$$+ n \sum_i \sum_j \sum_k (IJK)_{ijk}^2 + \sum_i \sum_j \sum_k \sum_l E_{ijkl}^2$$

$$= SS_A + SS_B + SS_C + SS_{AB} + SS_{AC} + SS_{BC} + SS_{ABC} + SS_E$$

$I_i = \widehat{\tau}_i \qquad J_j = \widehat{\beta}_j \qquad K_k = \widehat{\gamma}_k \qquad (IJ)_{ij} = \widehat{\tau\beta}_{ij} \qquad (IK)_{ik} = \widehat{\tau\gamma}_{ik}$

$(JK)_{jk} = \widehat{\beta\gamma}_{jk} \qquad (IJK)_{ijk} = \widehat{\tau\beta\gamma}_{ijk} \qquad E_{ijkl} = e_{ijkl}$

Three-Factor Factorial Designs: Fixed Factors A, B, C

CASE 1: $\ell \geq 2$ WE CAN CONSIDER THE ENTIRE MODEL

$$y_{ijk\ell} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\tau\gamma)_{ik} + (\beta\gamma)_{jk} + (\tau\beta\gamma)_{ijk} + \varepsilon_{ijk\ell}$$

- TEST HIGHEST ORDER INTERACTIONS FIRST.

  $H_0: (\tau\beta\gamma)_{ijk} = 0$ FOR ALL ijk VS $H_A: (\tau\beta\gamma)_{ijk} \neq 0$ FOR SOME ijk

- IF NOT SIGNIFICANT THEN CONSIDER THE SECOND ORDER INTERACTIONS

  $H_0: (\tau\beta)_{ij} = 0$ FOR ALL ij VS $H_A: (\tau\beta)_{ij} \neq 0$ FOR SOME ij

  $H_0: (\tau\gamma)_{ik} = 0$ FOR ALL ik VS $H_A: (\tau\gamma)_{ik} \neq 0$ FOR SOME ik

  $H_0: (\beta\gamma)_{jk} = 0$ FOR ALL jk VS $H_A: (\beta\gamma)_{jk} \neq 0$ FOR SOME jk

- IF NO SIGNIFICANT INTERACTION CONTAINS

  (1) SUBSCRIPT i (FACTOR A) THEN CONSIDER TESTING

  $H_0: \tau_i = 0$ FOR ALL i VS $H_A: \tau_i \neq 0$ FOR SOME i

  (2) SUBSCRIPT j (FACTOR B) THEN CONSIDER TESTING

  $H_0: \beta_j = 0$ FOR ALL j VS $H_A: \beta_j \neq 0$ FOR SOME j

  (3) SUBSCRIPT k (FACTOR C) THEN CONSIDER TESTING

  $H_0: \gamma_k = 0$ FOR ALL k VS $H_A: \gamma_k \neq 0$ FOR SOME k

CASE 2: $\ell = 1$ WE CANNOT CONSIDER THE $(\tau\beta\gamma)_{ijk}$ TERMS IN THE MODEL BECAUSE THERE ARE NO D.F. FOR ERROR. IN THESE CASES WE ASSUME $(\tau\beta\gamma)_{ijk} = 0$ FOR ALL ijk, THAT IS, THERE IS NO THREE-WAY INTERACTION; THEN THE D.F. ARE ATTRIBUTED TO ERROR. AND WE BEGIN TESTING WITH TWO-WAY INTERACTIONS.

IN ACTUALITY, THE MS_E CONTAINS BOTH THE RANDOM ERROR PLUS ANY THREE-WAY INTERACTION (NO MATTER HOW SMALL) EFFECTS.

Three Factor Factorial Example

- In a paper production process, the effects of percentage of hardwood concentration in raw wood pulp, the vat pressure, and the cooking time on the paper strength were studied.
- There were $a = 3$ levels of hardwood concentration (CONC = 2%, 4%, 8%).

  There were $b = 3$ levels of vat pressure (PRESS = 400, 500, 650).

  There were $c = 2$ levels of cooking time (TIME = 3 hours, 4 hours).

- A three factor factorial experiment with $n = 2$ replicates was run. The order of data collection was completely randomized.
- We assume all three factors are fixed.
- The experimental data are in the table below.

Hardwood Concentration	Cooking time 3.0 hours			Cooking time 4.0 hours		
	Pressure			Pressure		
	400	500	650	400	500	650
2	196.6	197.7	199.8	198.4	199.6	200.6
	196.0	196.0	199.4	198.6	200.4	200.9
4	198.5	196.0	198.4	197.5	198.7	199.6
	197.2	196.9	197.6	198.1	198.0	199.0
6	197.5	195.6	197.4	197.6	197.0	198.5
	196.6	196.2	198.1	198.4	197.8	199.8

- From the SAS output, the p-value = .2903 is not significant for the test of the equality of the three factor interaction effects: $\text{H}_0 : \tau\beta\gamma_{ijk} = 0$ for all i, j, k vs $\text{H}_1 : \tau\beta\gamma_{ijk} \neq 0$ for some i, j, k
- The p-values of .0843, .0146, and .0750 for the CONC*TIME, CONC*PRESS, and TIME*PRESS two factor interactions are all significant at the $\alpha = .10$ level indicating the interpretation of the significant main effects for CONC (p-value=.0009), TIME (p-value$< .0001$), and PRESS (p-value$< .0001$) may be masked. To understand the effects in the model, we need to examine the interaction plots.

THREE FACTOR ANALYSIS OF VARIANCE

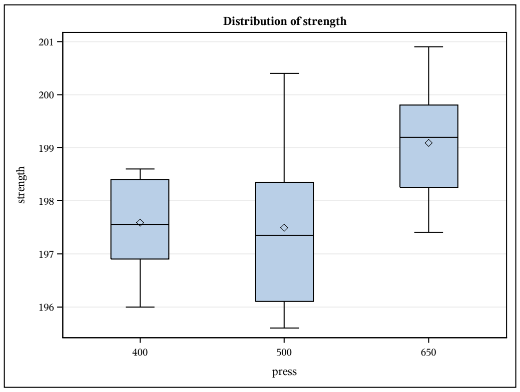
The GLM Procedure

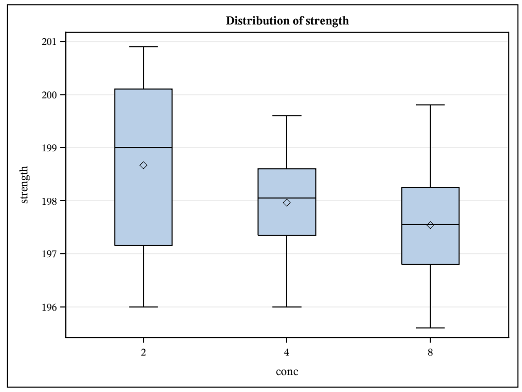
Variable: strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	17	59.72888889	3.51346405	9.61	<.0001
Error	18	6.58000000	0.36555556		
Corrected Total	35	66.30888889			

R-Square	Coeff Var	Root MSE	strength Mean
0.900767	0.305274	0.604612	198.0556

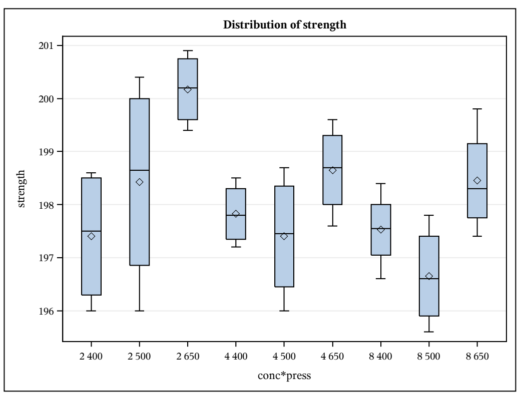
Source	DF	Type III SS	Mean Square	F Value	Pr > F
conc	2	7.76388889	3.88194444	10.62	0.0009
time	1	20.25000000	20.25000000	55.40	<.0001
conc*time	2	2.08166667	1.04083333	2.85	0.0843
press	2	19.37388889	9.68694444	26.50	<.0001
conc*press	4	6.09111111	1.52277778	4.17	0.0146
time*press	2	2.19500000	1.09750000	3.00	0.0750
conc*time*press	4	1.97333333	0.49333333	1.35	0.2903

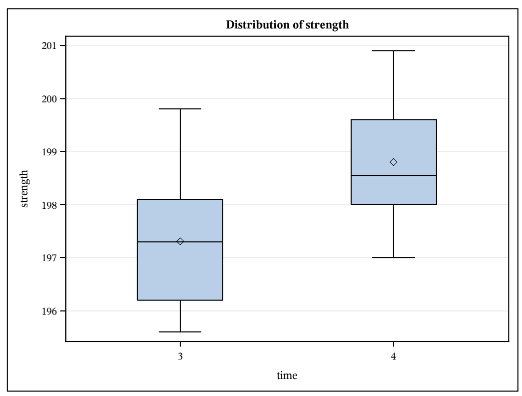




Level of press	N	strength	
		Mean	Std Dev
400	12	197.583333	0.85687947
500	12	197.491667	1.50903843
650	12	199.091667	1.12043687

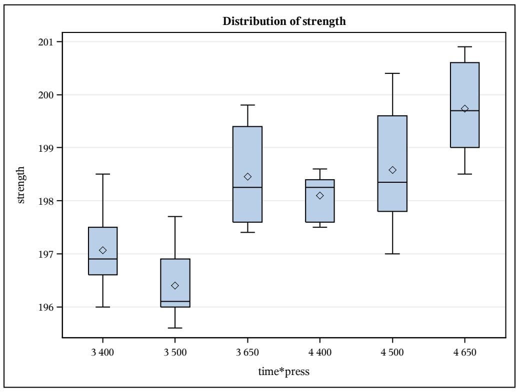
Level of conc	N	strength	
		Mean	Std Dev
2	12	198.666667	1.75620113
4	12	197.958333	0.98669175
8	12	197.541667	1.12448641

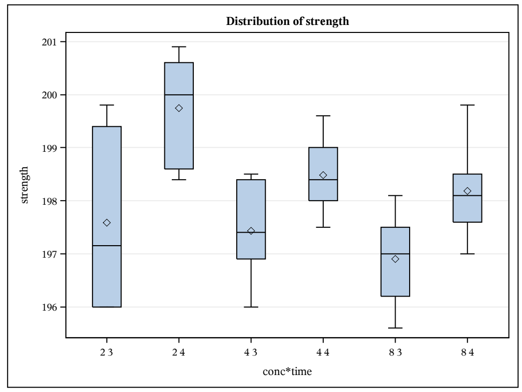




Level of conc	Level of press	N	strength	
			Mean	Std Dev
2	**400**	4	197.400000	1.29614814
2	**500**	4	198.425000	1.97378655
2	**650**	4	200.175000	0.69462220
4	**400**	4	197.825000	0.58523500
4	**500**	4	197.400000	1.19163753
4	**650**	4	198.650000	0.85440037
8	**400**	4	197.525000	0.73654599
8	**500**	4	196.650000	0.95742711
8	**650**	4	198.450000	1.00829890

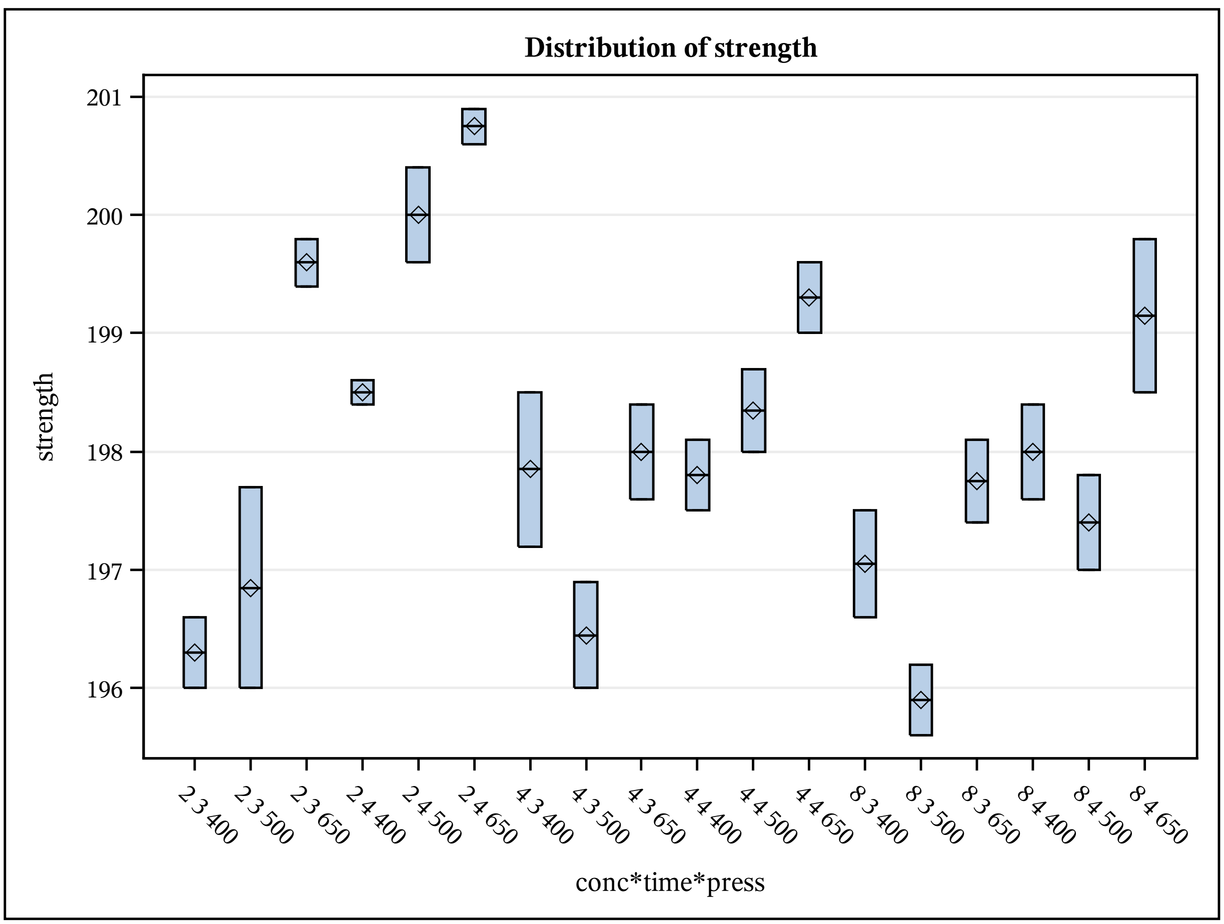
Level of time	N	strength	
		Mean	Std Dev
3	18	197.305556	1.20219027
4	18	198.805556	1.12431533





Level of time	Level of press	N	strength Mean	Std Dev
3	400	6	197.066667	0.87559504
3	500	6	196.400000	0.76681158
3	650	6	198.450000	0.96695398
4	400	6	198.100000	0.45607017
4	500	6	198.583333	1.24966662
4	650	6	199.733333	0.91578746

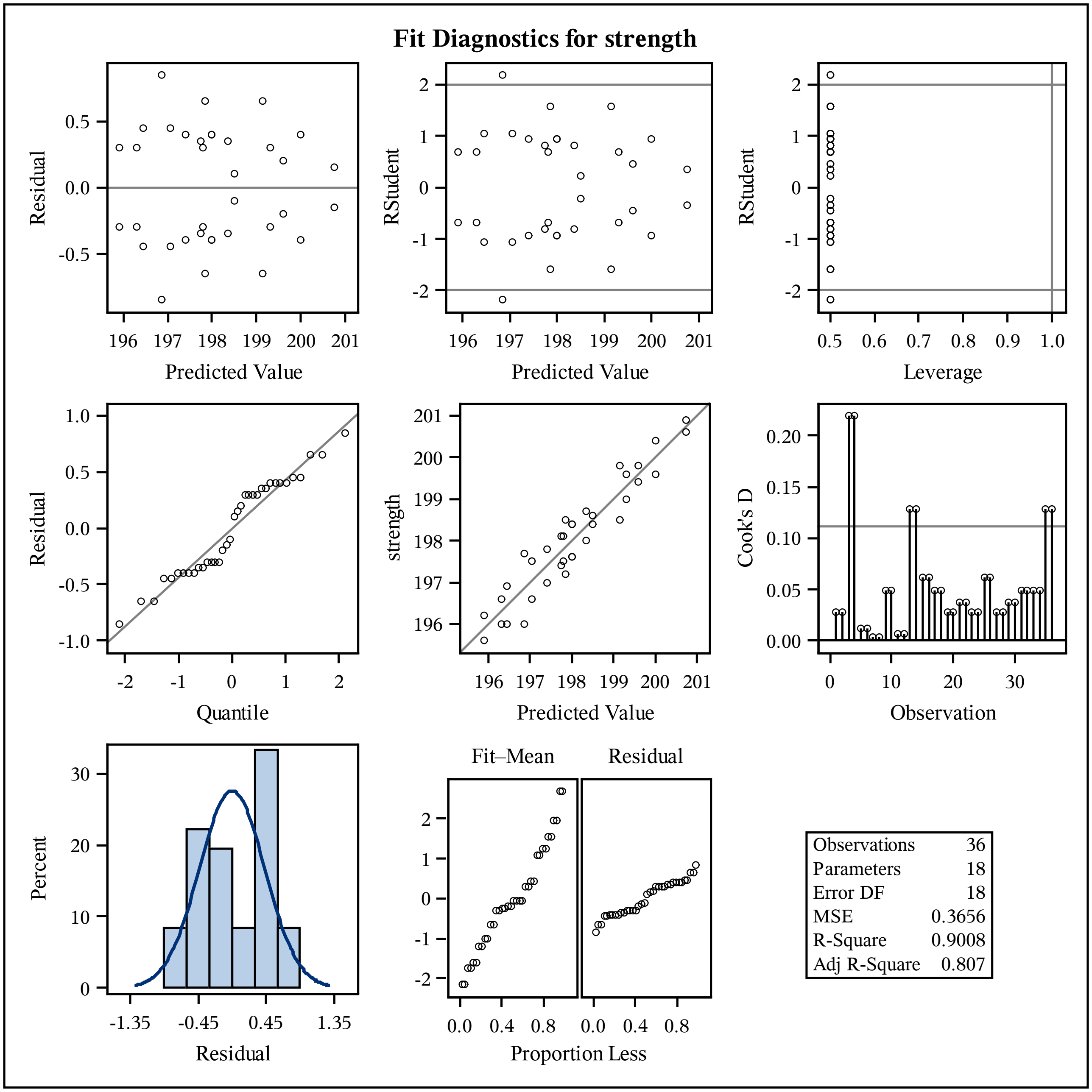
Level of conc	Level of time	N	strength Mean	Std Dev
2	3	6	197.583333	1.68572437
2	4	6	199.750000	1.06160256
4	3	6	197.433333	0.94798031
4	4	6	198.483333	0.76267075
8	3	6	196.900000	0.92951600
8	4	6	198.183333	0.96419224



Check of Normality Assumption

The UNIVARIATE Procedure
Variable: resid

Tests for Normality				
Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.938963	**Pr < W**	0.0472
Kolmogorov-Smirnov	**D**	0.172166	**Pr > D**	<0.0100
Cramer-von Mises	**W-Sq**	0.209114	**Pr > W-Sq**	<0.0050
Anderson-Darling	**A-Sq**	1.090312	**Pr > A-Sq**	0.0068



SAS Code for Three-Factor Factorial Example

```
****************************************;
*** THREE FACTOR ANALYSIS OF VARIANCE ***;
****************************************;

DATA in;
     DO conc = 2 , 4 , 8;
     DO time = 3 TO 4;
     DO press= 400 , 500 , 650;
     DO rep = 1 TO 2;
        INPUT strength @@;
        strength=strength + 190;  OUTPUT;
     END; END; END; END;
CARDS;
6.6 6.0   7.7 6.0   9.8 9.4   8.4 8.6   9.6 10.4   10.6 10.9
8.5 7.2   6.0 6.9   8.4 7.6   7.5 8.1   8.7  8.0    9.6  9.0
7.5 6.6   5.6 6.2   7.4 8.1   7.6 8.4   7.0  7.8    8.5  9.8

PROC GLM DATA=in PLOTS=(ALL);
     CLASS conc time press;
     MODEL strength = conc|time|press / SS3;
   * MEANS press|conc|time@2;
     MEANS press|conc|time;
     OUTPUT OUT=diag R=resid;
TITLE 'THREE FACTOR ANALYSIS OF VARIANCE';

PROC UNIVARIATE DATA=diag NORMAL PLOTS;
     VAR resid;
TITLE "Check of Normality Assumption";
RUN;
```

4.16 Two Factor Factorial ANOVA with Blocks

<u>EXAMPLE</u>: The yield of a chemical process is being studied.

- The two factors of interest are Temperature and Pressure. Three levels of each factor ($a = 3$, $b = 3$) are selected.
- Only nine runs can be made in one day. The experimenter runs a complete replicate of the two factor factorial design on each day.
- The data are shown in the following table. Analyze the data assuming that the days are blocks.
- You just have to add a block effect (DAY) to the two factor factorial model.

	Day 1			Day 2		
	Pressure			Pressure		
Temperature	250	260	270	250	260	270
Low	86.3	84.0	85.8	86.1	85.2	87.3
Medium	88.5	87.3	89.0	89.4	89.9	90.3
High	89.1	90.2	91.3	91.7	93.2	93.7

TWO-FACTOR FACTORIAL WITH BLOCKS

The GLM Procedure

Variable: yield

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	9	122.8194444	13.6466049	25.69	<.0001
Error	8	4.2500000	0.5312500		
Corrected Total	17	127.0694444			

R-Square	Coeff Var	Root MSE	yield Mean
0.966554	0.820850	0.728869	88.79444

Source	DF	Type III SS	Mean Square	F Value	Pr > F
temp	2	99.85444444	49.92722222	93.98	<.0001
press	2	5.50777778	2.75388889	5.18	0.0360
temp*press	4	4.45222222	1.11305556	2.10	0.1733
day	1	13.00500000	13.00500000	24.48	0.0011

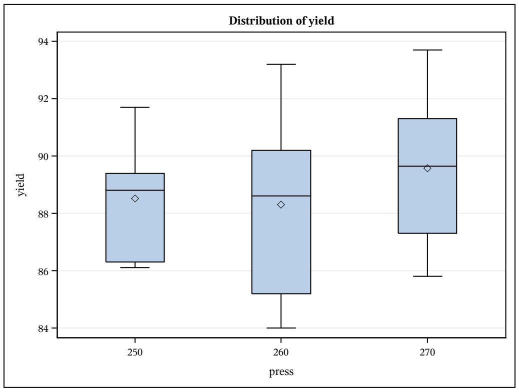
Source	Type III Expected Mean Square
temp	Var(Error) + Q(temp,temp*press)
press	Var(Error) + Q(press,temp*press)
temp*press	Var(Error) + Q(temp*press)
day	Var(Error) + 9 Var(day)

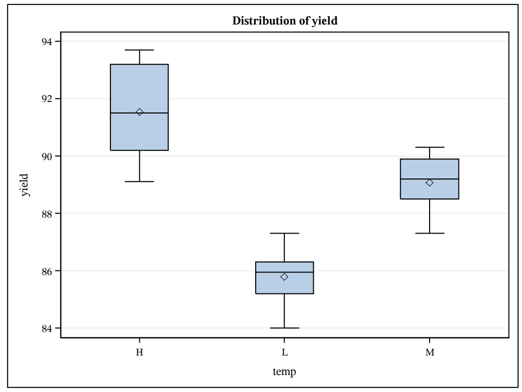
The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance

Variable: yield

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	**temp**	2	99.854444	49.927222	93.98	<.0001
*	**press**	2	5.507778	2.753889	5.18	0.0360
	temp*press	4	4.452222	1.113056	2.10	0.1733
	day	1	13.005000	13.005000	24.48	0.0011
	Error: MS(Error)	8	4.250000	0.531250		

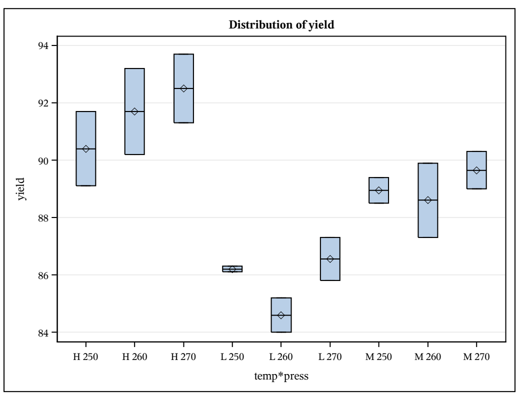
*** This test assumes one or more other fixed effects are zero.**

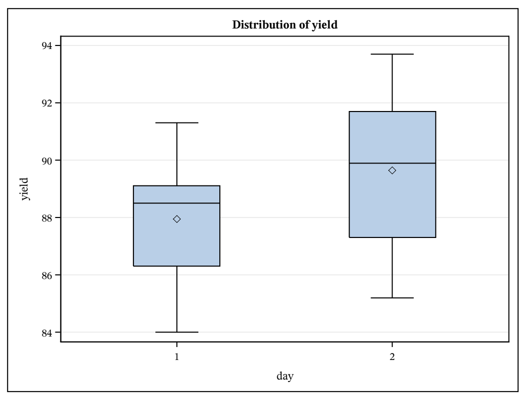




Level of press	N	yield Mean	Std Dev
250	6	88.5166667	2.09801493
260	6	88.3000000	3.44325427
270	6	89.5666667	2.83807446

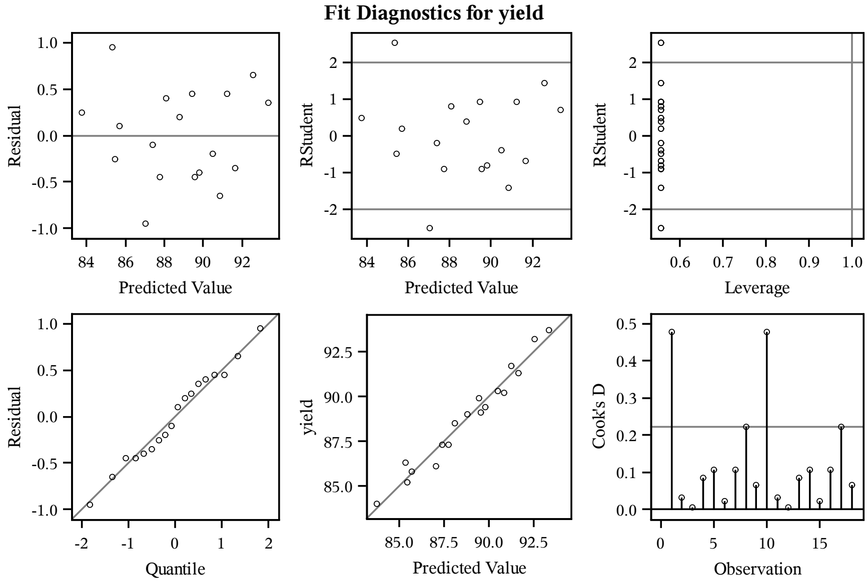
Level of temp	N	yield Mean	Std Dev
H	6	91.5333333	1.74661578
L	6	85.7833333	1.11250468
M	6	89.0666667	1.07455417





Level of temp	Level of press	N	yield Mean	Std Dev
H	250	2	90.4000000	1.83847763
H	260	2	91.7000000	2.12132034
H	270	2	92.5000000	1.69705627
L	250	2	86.2000000	0.14142136
L	260	2	84.6000000	0.84852814
L	270	2	86.5500000	1.06066017
M	250	2	88.9500000	0.63639610
M	260	2	88.6000000	1.83847763
M	270	2	89.6500000	0.91923882

Level of day	N	yield Mean	Std Dev
1	9	87.9444444	2.30169020
2	9	89.6444444	2.99337231



TWO-FACTOR FACTORIAL WITH BLOCKS
ANALYSIS OF MODEL WITH TWO-FACTOR BLOCK INTERACTIONS

The GLM Procedure

Variable: yield

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	13	126.3861111	9.7220085	56.91	0.0007
Error	4	0.6833333	0.1708333		
Corrected Total	17	127.0694444			

R-Square	Coeff Var	Root MSE	yield Mean
0.994622	0.465479	0.413320	88.79444

Source	DF	Type III SS	Mean Square	F Value	Pr > F
temp	2	99.85444444	49.92722222	292.26	<.0001
press	2	5.50777778	2.75388889	16.12	0.0122
temp*press	4	4.45222222	1.11305556	6.52	0.0484
day	1	13.00500000	13.00500000	76.13	0.0010
day*temp	2	2.54333333	1.27166667	7.44	0.0448
day*press	2	1.02333333	0.51166667	3.00	0.1603

Source	Type III Expected Mean Square
temp	Var(Error) + 3 Var(day*temp) + Q(temp,temp*press)
press	Var(Error) + 3 Var(day*press) + Q(press,temp*press)
temp*press	Var(Error) + Q(temp*press)
day	Var(Error) + 3 Var(day*press) + 3 Var(day*temp) + 9 Var(day)
day*temp	Var(Error) + 3 Var(day*temp)
day*press	Var(Error) + 3 Var(day*press)

TWO-FACTOR FACTORIAL WITH BLOCKS
ANALYSIS OF MODEL WITH TWO-FACTOR BLOCK INTERACTIONS

The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance

Variable: yield

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	temp	2	99.854444	49.927222	39.26	0.0248
	Error: MS(day*temp)	2	2.543333	1.271667		
* This test assumes one or more other fixed effects are zero.						

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	press	2	5.507778	2.753889	5.38	0.1567
	Error: MS(day*press)	2	1.023333	0.511667		
* This test assumes one or more other fixed effects are zero.						

Source	DF	Type III SS	Mean Square	F Value	Pr > F
temp*press	4	4.452222	1.113056	6.52	0.0484
day*temp	2	2.543333	1.271667	7.44	0.0448
day*press	2	1.023333	0.511667	3.00	0.1603
Error: MS(Error)	4	0.683333	0.170833		

Source	DF	Type III SS	Mean Square	F Value	Pr > F
day	1	13.005000	13.005000	8.07	0.0728
Error	2.7464	4.428501	1.612500		
Error: MS(day*temp) + MS(day*press) - MS(Error)					

SAS Code

```
DM 'LOG; CLEAR; OUT; CLEAR;';

ODS GRAPHICS ON;
ODS PRINTER PDF file='C:\COURSES\ST541\TWOBLOCK.PDF';
OPTIONS NODATE NONUMBER;

****************************************;
*** TWO-FACTOR FACTORIAL WITH BLOCKS ***;
****************************************;

DATA in;
     DO day = 1 TO 2;
     DO temp = 'L' , 'M' , 'H';
     DO press= 250 TO 270 BY 10;
        INPUT yield @@;  OUTPUT;
     END; END; END;
CARDS;
86.3 84.0 85.8  88.5 87.3 89.0  89.1 90.2 91.3
86.1 85.2 87.3  89.4 89.9 90.3  91.7 93.2 93.7

PROC GLM DATA=in PLOTS=(ALL);
     CLASS day temp press;
     MODEL yield = temp|press day / SS3;
     MEANS press|temp day;
```

```
     RANDOM day / TEST;
     OUTPUT OUT=diag P=pred R=resid;
TITLE 'TWO-FACTOR FACTORIAL WITH BLOCKS';

PROC UNIVARIATE DATA=diag NORMAL ;
     VAR resid;

*** MODEL INCLUDING BLOCK INTERACTIONS ***;

PROC GLM DATA=in;
     CLASS day temp press;
     MODEL yield = temp|press|day@2 / SS3;
     RANDOM day day*temp day*press / TEST;
TITLE2 'ANALYSIS OF MODEL WITH TWO-FACTOR BLOCK INTERACTIONS';
RUN;
```

4.17 Expected Means Squares (EMS) Algorithm for Balanced Designs (Supplemental)

- Classify each effect as either a fixed effect or a random effect.
- A fixed effect is represented by the EMS component

$$\frac{\sum(\text{fixed effect})^2}{\text{d.f. for the fixed effect}} \qquad \text{For example: } \frac{\sum_{i=1}^{a}\tau_i^2}{a-1}$$

- A random effect is represented by a variance component with its associated subscripts. For example: σ^2_τ.
- An interaction containing at least one random effect is considered random and is represented by a variance component. For example:
  - If factor A is random and factor B is random, then the A*B interaction is random. The associated variance component is $\sigma^2_{\tau\beta}$.
  - If factor A is fixed and factor B is random, then the A*B interaction is random. The associated variance component is $\sigma^2_{\tau\beta}$.
  - If factor A is fixed with a levels and factor B is fixed with b levels, then the A*B interaction is fixed. The associated EMS component is $\dfrac{\sum_{i=1}^{a}\sum_{i=1}^{b}\tau\beta_{ij}^2}{(a-1)(b-1)}$.

STEP 1: Prepare the EMS Table

1. Set up a row for each model effect that has one or more subscripts. Write the error term $\epsilon_{ij\ldots m}$ as a 'nested' effect $\epsilon_{m(ij\ldots)}$. We say that $\epsilon_{m(ij\ldots)}$ is the m^{th} replicate for treatment combination $ij\ldots$. We will use parentheses () to indicate which effects are nested. Nested effects will be discussed in detail later in the course.
2. Set up columns for each subscript. Above each subscript, write the number of levels associated with the factor and whether the factor is fixed (F) or random (R).
3. For each row effect, subscripts are divided into three classes: live, dead, or absent.
   - A subscript is **live** if it is present and is not in parentheses.
   - A subscript is **dead** if it is present and is in parentheses.
   - A subscript is **absent** if it is not present.

Example of Step 1: Consider the two-factor factorial design. Factor A is fixed with a levels and factor B is random with b levels. n replicates were taken for each of the ab combinations of the levels of A and B. We will be using the mixed model $y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \epsilon_{ijk}$.

		F	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$				
β_j	σ^2_β				
$\alpha\beta_{ij}$	$\sigma^2_{\alpha\beta}$				
$\epsilon_{k(ij)}$	σ^2				

STEP 2: Filling in the rows of the EMS Table:

1. Write 1 in each column containing dead subscripts.

		F	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$				
β_j	σ^2_β				
$\alpha\beta_{ij}$	$\sigma^2_{\alpha\beta}$				
$\epsilon_{k(ij)}$	σ^2	1	1		

2. If any row subscript corresponds to a random factor (R), then write 1 in all columns with a matching subscript. Otherwise, write 0 in all columns with a matching subscript.

		F	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$	0			
β_j	σ^2_β		1		
$\alpha\beta_{ij}$	$\sigma^2_{\alpha\beta}$	1	1		
$\epsilon_{k(ij)}$	σ^2	1	1	1	

3. For the remaining missing values, enter the number of factor levels for that column.

		F	R	R	
		a	b	n	EMS
Effect	Component	i	j	k	
α_i	$\sum \alpha_i^2/(a-1)$	0	b	n	
β_j	σ^2_β	a	1	n	
$\alpha\beta_{ij}$	$\sigma^2_{\alpha\beta}$	1	1	n	
$\epsilon_{k(ij)}$	σ^2	1	1	1	

STEP 3: Obtaining the EMS

1. Consider all rows containing all of the subscripts in the row effect.
2. For each row, delete all columns containing live subscripts. Take the product of the remaining values and the component.
3. EMS = the sum of these products.

In the example, for α_i, there is only one subscript i. In the model α_i, $\alpha\beta_{ij}$, and $\epsilon_{k(ij)}$ all contain subscript i. We delete the i column and generate the three products

(a) $\dfrac{bn\sum \alpha_i^2}{a-1}$ from the α_i row (b) $n\sigma^2_{\alpha\beta}$ from the $\alpha\beta_{ij}$ row (c) σ^2 from the $\epsilon_{k(ij)}$ row.

Effect	Component	F a i	R b j	R n k	EMS
α_i	$\sum \alpha_i^2/(a-1)$	0	b	n	$\sigma^2 + n\sigma^2_{\alpha\beta} + \dfrac{bn\sum \alpha_i^2}{a-1}$
β_j	σ^2_β	a	1	n	
$\alpha\beta_{ij}$	$\sigma^2_{\alpha\beta}$	1	1	n	
$\epsilon_{k(ij)}$	σ^2	1	1	1	

In the example, for β_j, there is only one subscript j. In the model β_j, $\alpha\beta_{ij}$, and $\epsilon_{k(ij)}$ all contain subscript j. We delete the j column and generate the three products

(a) $an\sigma^2_\beta$ from the β_j row, (b) $n\sigma^2_{\alpha\beta}$ from the $\alpha\beta_{ij}$ row, (c) σ^2 from the $\epsilon_{k(ij)}$ row.

Effect	Component	F a i	R b j	R n k	EMS
α_i	$\sum \alpha_i^2/(a-1)$	0	b	n	$\sigma^2 + n\sigma^2_{\alpha\beta} + \dfrac{bn\sum \alpha_i^2}{a-1}$
β_j	σ^2_β	a	1	n	$\sigma^2 + n\sigma^2_{\alpha\beta} + an\sigma^2_\beta$
$\alpha\beta_{ij}$	$\sigma^2_{\alpha\beta}$	1	1	n	
$\epsilon_{k(ij)}$	σ^2	1	1	1	

In the example, for $\alpha\beta_{ij}$, there are two subscripts i and j. In the model $\alpha\beta_{ij}$ and $\epsilon_{k(ij)}$ all contain subscripts i and j. We delete the i and j columns and generate the two products

(a) $n\sigma^2_{\alpha\beta}$ from the $\alpha\beta_{ij}$ row and (b) σ^2 from the $\epsilon_{k(ij)}$ row.

Effect	Component	F a i	R b j	R n k	EMS
α_i	$\sum \alpha_i^2/(a-1)$	0	b	n	$\sigma^2 + n\sigma^2_{\alpha\beta} + \dfrac{bn\sum \alpha_i^2}{a-1}$
β_j	σ^2_β	a	1	n	$\sigma^2 + n\sigma^2_{\alpha\beta} + an\sigma^2_\beta$
$\alpha\beta_{ij}$	$\sigma^2_{\alpha\beta}$	1	1	n	$\sigma^2 + n\sigma^2_{\alpha\beta}$
$\epsilon_{k(ij)}$	σ^2	1	1	1	

Finally, for $\epsilon_{k(ij)}$, there are three subscripts i, j, and k. In the model only $\epsilon_{k(ij)}$ contains all three subscripts. We delete the i, j, and k columns. This leaves only σ^2.

Effect	Component	F a i	R b j	R n k	EMS
α_i	$\sum \alpha_i^2/(a-1)$	0	b	n	$\sigma^2 + n\sigma^2_{\alpha\beta} + \dfrac{bn\sum \alpha_i^2}{a-1}$
β_j	σ^2_β	a	1	n	$\sigma^2 + n\sigma^2_{\alpha\beta} + an\sigma^2_\beta$
$\alpha\beta_{ij}$	$\sigma^2_{\alpha\beta}$	1	1	n	$\sigma^2 + n\sigma^2_{\alpha\beta}$
$\epsilon_{k(ij)}$	σ^2	1	1	1	σ^2

Next consider the three-factor factorial design for which all three factors are random. The number of factor levels are a, b, and c with n replicates taken for each three-factor treatment combination. The full three factor-factorial model is

$$y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + \tau\beta_{ij} + \tau\gamma_{ik} + \beta\gamma_{jk} + \tau\beta\gamma_{ijk} + \epsilon_{ijkl}$$

From Step 1 and Step 2, we have

		R	R	R	R	
		a	b	c	n	EMS
Effect	Component	i	j	k	l	
τ_i	σ^2_τ	1				
β_j	σ^2_β		1			
γ_k	σ^2_γ			1		
$\tau\beta_{ij}$	$\sigma^2_{\tau\beta}$	1	1			
$\tau\gamma_{ik}$	$\sigma^2_{\tau\gamma}$	1		1		
$\beta\gamma_{jk}$	$\sigma^2_{\beta\gamma}$		1	1		
$\tau\beta\gamma_{ijk}$	$\sigma^2_{\tau\beta\gamma}$	1	1	1		
$\epsilon_{l(ijk)}$	σ^2	1	1	1	1	

		R	R	R	R	
		a	b	c	n	EMS
Effect	Component	i	j	k	l	
τ_i	σ^2_τ	1	b	c	n	
β_j	σ^2_β	a	1	c	n	
γ_k	σ^2_γ	a	b	1	n	
$\tau\beta_{ij}$	$\sigma^2_{\tau\beta}$	1	1	c	n	
$\tau\gamma_{ik}$	$\sigma^2_{\tau\gamma}$	1	b	1	n	
$\beta\gamma_{jk}$	$\sigma^2_{\beta\gamma}$	a	1	1	n	
$\tau\beta\gamma_{ijk}$	$\sigma^2_{\tau\beta\gamma}$	1	1	1	n	
$\epsilon_{l(ijk)}$	σ^2	1	1	1	1	

		R	R	R	R	
		a	b	c	n	EMS
Effect	Component	i	j	k	l	
τ_i	σ^2_τ	1	b	c	n	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + bn\sigma^2_{\tau\gamma} + cn\sigma^2_{\tau\beta} + bcn\sigma^2_\tau$
β_j	σ^2_β	a	1	c	n	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + an\sigma^2_{\beta\gamma} + cn\sigma^2_{\tau\beta} + acn\sigma^2_\beta$
γ_k	σ^2_γ	a	b	1	n	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + an\sigma^2_{\beta\gamma} + bn\sigma^2_{\tau\gamma} + abn\sigma^2_\gamma$
$\tau\beta_{ij}$	$\sigma^2_{\tau\beta}$	1	1	c	n	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + cn\sigma^2_{\tau\beta}$
$\tau\gamma_{ik}$	$\sigma^2_{\tau\gamma}$	1	b	1	n	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + bn\sigma^2_{\tau\gamma}$
$\beta\gamma_{jk}$	$\sigma^2_{\beta\gamma}$	a	1	1	n	$\sigma^2 + n\sigma^2_{\tau\beta\gamma} + an\sigma^2_{\beta\gamma}$
$\tau\beta\gamma_{ijk}$	$\sigma^2_{\tau\beta\gamma}$	1	1	1	n	$\sigma^2 + n\sigma^2_{\tau\beta\gamma}$
$\epsilon_{l(ijk)}$	σ^2	1	1	1	1	σ^2

- This is the simplest example when no ratio of EMS exists for defining F-statistics to test certain null hypotheses. In all of our previous examples, when H_0 is true

$$\text{E(numerator MS)} \ = \ \text{E(denominator MS)}.$$

- For this example, We have exact tests for

  (1) $\text{H}_0 : \sigma^2_{\beta\gamma} = 0$,

  (2) $\text{H}_0 : \sigma^2_{\tau\gamma} = 0$, and

  (3) $\text{H}_0 : \sigma^2_{\tau\beta} = 0$

  if we use $MS_{\tau\beta\gamma}$ as the denominator of the F-statistic. We also have an exact test for

  (4) $\text{H}_0 : \sigma^2_{\tau\beta\gamma} = 0$ if we use MS_E as the denominator of the F-statistic.

- We do not have exact tests for

  (5) $\text{H}_0 : \sigma^2_{\tau} = 0$,

  (6) $\text{H}_0 : \sigma^2_{\beta} = 0$, and

  (7) $\text{H}_0 : \sigma^2_{\gamma} = 0$

  because when (5), (6), or (7) is true, there is no matching EMS. Thus, there is no exact F-test.

4.18 Approximate F-Tests

- If we want to make inferences about effects for which there is no exact F-test, we can use **approximate F-tests**, a procedure attributed to Sattherthwaite (1946).
- In this procedure, the goal is to find linear combinations of EMS that are equal when H_0 is true. That is, find

$$MS' = MS_r + \cdots + MS_s \qquad \text{and} \qquad MS'' = MS_u + \cdots + MS_v$$

  such the $E(MS') - E(MS'')$ equals the effect of interest.

- The test statistic would be $F = \dfrac{MS'}{MS''}$ which is approximately distributed as $F(p, q)$ where the degrees of freedom p and q are

$$p = \frac{(MS_r + \cdots + MS_s)^2}{MS_r^2/f_r + \cdots + MS_s^2/f_s} \qquad \text{and} \qquad q = \frac{(MS_u + \cdots + MS_v)^2}{MS_u^2/f_u + \cdots + MS_v^2/f_v}.$$

  f_i is the degrees of freedom associated with mean square MS_i. It is unlikely that p and q are integers.

- Let the A, B, and C be the factors in a three-factor factorial design, and A, B, and C are random. Consider the following:

$$MS' = MS_A + MS_{ABC} \qquad \text{and} \qquad MS'' = MS_{AB} + MS_{AC}$$

  Then, $E(MS') - E(MS'')$

$$=$$

- Therefore, to test $\text{H}_0 : \sigma^2_\tau = 0$, we can use $F = \dfrac{MS'}{MS''}$ with d.f.

$$p = \frac{(MS_A + MS_{ABC})^2}{\frac{MS^2_A}{a-1} + \frac{MS^2_{ABC}}{(a-1)(b-1)(c-1)}} \qquad \text{and} \qquad q = \frac{(MS_{AB} + MS_{AC})^2}{\frac{MS^2_{AB}}{(a-1)(b-1)} + \frac{MS^2_{AC}}{(a-1)(c-1)}}.$$

- Note that when $\text{H}_0 : \sigma^2_\tau = 0$ is true, $E(MS') = E(MS'')$.
- Note also the choice of MS' and MS'' is not unique. For example, suppose we define

$$MS' = MS_A \qquad \text{and} \qquad MS'' = MS_{AB} + MS_{AC} - MS_{ABC}$$

Then,

$$\begin{aligned} E(MS') - E(MS'') &= \sigma^2 + n\sigma^2_{\tau\beta\gamma} + bn\sigma^2_{\tau\gamma} + cn\sigma^2_{\tau\beta} + bcn\sigma^2_\tau \\ &\quad - \left[\sigma^2 + n\sigma^2_{\tau\beta\gamma} + bn\sigma^2_{\tau\gamma} + cn\sigma^2_{\tau\beta}\right] \\ &= bcn\sigma^2_\tau \end{aligned}$$

- Therefore, to test $\text{H}_0 : \sigma^2_\tau = 0$, we can use $F = \dfrac{MS'}{MS''}$ with d.f.

$$p = \frac{(MS_A)^2}{\frac{MS^2_A}{a-1}} = a - 1 \qquad \text{and} \qquad q = \frac{(MS_{AB} + MS_{AC} - MS_{ABC})^2}{\frac{MS^2_{AB}}{(a-1)(b-1)} + \frac{MS^2_{AC}}{(a-1)(c-1)} + \frac{MS^2_{ABC}}{(a-1)(b-1)(c-1)}}.$$

- Again, note that when $\text{H}_0 : \sigma^2_\tau = 0$ is true, $E(MS') = E(MS'')$.

4.18.1 Determining Exact and Approximate F-tests using SAS

- Prior to running an experiment that contains random effects, I recommend that you determine what exact tests and approximate F-tests can be performed.
- Once you have a design:
  1. Generate random responses using a random number generator.
  2. Analyze the random responses in SAS using the RANDOM statement with the /TEST opion.
  3. Examine the output. For each test of a model effect, see if the denominator of the F-statistics is a single mean square (exact F-test) or is a linear combination of means squares (approximate F-test).
- Consider the three factor factorial example and factors A, B, and C are random with $a = 3$, $b = 2$, $c = 2$, and $n = 3$.
  - There are exact tests for the $A * B$, $A * C$, $B * C$, and $A * B * C$ effects.
  - There are only approximate tests for the A, B, and C effects.

```
DATA in;
     DO A = 1 to 3;
     DO B = 1 to 2;
     DO C = 1 to 2;
     DO rep = 1 TO 3;
         MU = A + B + C;
        Y = MU + RANNOR(542057); OUTPUT;
     END; END; END; END;

PROC GLM DATA=in;
     CLASS A B C;
     MODEL Y = A|B|C / SS3;
     RANDOM A|B|C / TEST;
TITLE 'THE RANDOM EFFECTS MODEL WITH A, B, AND C RANDOM';
RUN;
```

THE RANDOM EFFECTS MODEL WITH A, B, AND C RANDOM

The GLM Procedure

Source	Type III Expected Mean Square
A	Var(Error) + 3 Var(A*B*C) + 6 Var(A*C) + 6 Var(A*B) + 12 Var(A)
B	Var(Error) + 3 Var(A*B*C) + 9 Var(B*C) + 6 Var(A*B) + 18 Var(B)
A*B	Var(Error) + 3 Var(A*B*C) + 6 Var(A*B)
C	Var(Error) + 3 Var(A*B*C) + 9 Var(B*C) + 6 Var(A*C) + 18 Var(C)
A*C	Var(Error) + 3 Var(A*B*C) + 6 Var(A*C)
B*C	Var(Error) + 3 Var(A*B*C) + 9 Var(B*C)
A*B*C	Var(Error) + 3 Var(A*B*C)

Tests of Hypotheses for Random Model Analysis of Variance

Variable: Y

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A	2	19.271207	9.635603	15.29	0.6679
Error	0.1521	0.095844	0.630278		
Error: MS(A*B) + MS(A*C) - MS(A*B*C) + 22E-17*MS(Error)					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
B	1	9.716443	9.716443	3.99	0.3872
Error	0.646	1.572278	2.433893		
Error: MS(A*B) + MS(B*C) - MS(A*B*C) + 11E-17*MS(Error)					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A*B	2	2.817828	1.408914	0.88	0.5319
A*C	2	1.644797	0.822398	0.51	0.6606
B*C	1	2.626013	2.626013	1.64	0.3287
Error: MS(A*B*C)	2	3.202069	1.601035		

Source	DF	Type III SS	Mean Square	F Value	Pr > F
C	1	21.373570	21.373570	11.57	0.4048
Error	0.4008	0.740359	1.847377		
Error: MS(A*C) + MS(B*C) - MS(A*B*C) + 11E-17*MS(Error)					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A*B*C	2	3.202069	1.601035	2.35	0.1171
Error: MS(Error)	24	16.361893	0.681746		

- Consider the three factor factorial example and factors A and B are fixed, and factor C is random with $a = 3$, $b = 2$, $c = 2$, and $n = 3$.
  - There are exact tests for the A, B, $A * B$, $A * C$, $B * C$, and $A * B * C$ effects.
  - There is an approximate test for the C effect.
- The 'Q' components in the EMS correspond to fixed effects.

```
PROC GLM DATA=in;
     CLASS A B C;
     MODEL Y = A|B|C / SS3;
     RANDOM C A*C B*C A*B*C / TEST;
TITLE 'THE RANDOM EFFECTS MODEL WITH A, B FIXED AND C RANDOM';
```

THE RANDOM EFFECTS MODEL WITH A, B FIXED AND C RANDOM

The GLM Procedure

Source	Type III Expected Mean Square
A	Var(Error) + 3 Var(A*B*C) + 6 Var(A*C) + Q(A,A*B)
B	Var(Error) + 3 Var(A*B*C) + 9 Var(B*C) + Q(B,A*B)
A*B	Var(Error) + 3 Var(A*B*C) + Q(A*B)
C	Var(Error) + 3 Var(A*B*C) + 9 Var(B*C) + 6 Var(A*C) + 18 Var(C)
A*C	Var(Error) + 3 Var(A*B*C) + 6 Var(A*C)
B*C	Var(Error) + 3 Var(A*B*C) + 9 Var(B*C)
A*B*C	Var(Error) + 3 Var(A*B*C)

Tests of Hypotheses for Mixed Model Analysis of Variance

Variable: Y

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	A	2	19.271207	9.635603	11.72	0.0786
	Error: MS(A*C)	2	1.644797	0.822398		
* This test assumes one or more other fixed effects are zero.						

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	B	1	9.716443	9.716443	3.70	0.3052
	Error: MS(B*C)	1	2.626013	2.626013		
* This test assumes one or more other fixed effects are zero.						

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A*B	2	2.817828	1.408914	0.88	0.5319
A*C	2	1.644797	0.822398	0.51	0.6606
B*C	1	2.626013	2.626013	1.64	0.3287
Error: MS(A*B*C)	2	3.202069	1.601035		

Source	DF	Type III SS	Mean Square	F Value	Pr > F
C	1	21.373570	21.373570	11.57	0.4048
Error	0.4008	0.740359	1.847377		
Error: MS(A*C) + MS(B*C) - MS(A*B*C) + 11E-17*MS(Error)					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A*B*C	2	3.202069	1.601035	2.35	0.1171
Error: MS(Error)	24	16.361893	0.681746		