Foldover Designs

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The *average effect of a factor* is defined as the difference between the averages of the observations when that factor is at its respective high and low levels. The uppercase letter used to represent a factor is also used to represent its corresponding average effect. [4; 80-81, 92-94, 103-104] For example, the average effect of factor A in a 2^4 factorial design is defined as follows:

$$A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{a + ab + ac + abc + ad + abd + acd + abcd}{8n} - \frac{(1) + b + c + bc + d + bd + cd + bcd}{8n} =$$

$$\frac{1}{8n}\left[-(1) + a - b + ab - c + ac - bc + abc - d + ad - bd + abd - cd + acd - bcd + abcd\right] \quad (1)$$

The average effect of a factor may be referred to simply as its "effect". For instance, the average effect of factor A may be called "the effect of factor A", "the factor A effect", or "the A effect".

The two-factor interaction effect between any two factors is defined as one half of the difference between the average effects of the first when the second is at its respective high and low levels.[4; 81-82, 94-95, 103-104], [5; 313-314]* Two-factor and higher-order interaction effects are denoted by the sequence of uppercase letters (in alphabetical order) representing the factors involved.[4; 80-82, 92, 94-95, 103]* As an example, the AB interaction effect in a 2^4 factorial design is defined as

$$AB = \frac{1}{2}\left[\left(\frac{ab + abc + abd + abcd}{4n} - \frac{b + bc + bd + bcd}{4n}\right) - \left(\frac{a + ac + ad + acd}{4n} - \frac{(1) + c + d + cd}{4n}\right)\right] =$$

$$\frac{1}{8n}\left[(1) - a - b + ab + c - ac - bc + abc + d - ad - bd + abd + cd - acd - bcd + abcd\right] \quad (2)$$

* See the final paragraph in **1. Introduction.**

1. Introduction.

The purpose of this paper is to give a brief introduction to two-level factorial and fractional factorial designs followed by a discussion of fold-over designs. We conclude with two examples of the full fold-over technique using selected data from a 2^6 factorial experiment. The conclusions reached from the analysis of each simulated fold-over experiment are then compared with those from the analysis of the full 2^6 factorial experiment.

The reader should note that any given design discussed in this paper has the same number of replicates, n, taken at each of its factor level combinations. Furthermore, all factor levels discussed in this paper are fixed.[1; 1234]

Some statements in this paper of results that apply in general have been made at least in part based on results and/or comments pertaining to specific examples or cases addressed in the reference textbooks. This has been done when the example or case appears to have been used to illustrate the corresponding general result and an asterisk follows the citing of the reference in those circumstances.

2. 2^k Factorial Designs.

A 2^k *factorial design* involves k factors such that each has two levels. The name follows from the fact that each replication of the design consists of 2^k factor level combinations.[4; 79] Also, the factor level combinations are randomly allocated to the experimental units in these designs.[1; 1237]

Each factor in a 2^k factorial design is denoted by a different uppercase letter of the Latin alphabet. One of the levels of each factor is designated as the *low* level and the other as the *high* level with the choice of designation being inconsequential.[4; 79-80, 91, 103]

Any given run where at least one factor is at its high level is identified by a sequence of lowercase Latin letters. The presence of a lowercase letter in the sequence indicates that the factor denoted by the uppercase of that letter is at its high level. If a lowercase letter is not in the sequence, then the associated factor is at its low level. The treatment combination where no factor is at its high level is designated as "(1)". In addition to identifying a given run, the notation described above is also used to designate the sum of the replicates of that run. [4; 81, 91, 103] As an example, the sequence of letters bc in a 2^4 factorial design identifies the run where factors A and D are at their low levels and factors B and C are at their high levels. It also designates the sum of the replicates of this run.

Table 1, shown previously, provides important information regarding 2^4 factorial designs. Note that the treatment combinations in the "Run" column there have been listed in *standard order*. Standard order in a 2^k factorial design consists of placing the run where all factors are at their low levels first followed by the run where only factor A is at its high level. Next listed is the run where only factor B is at its high level followed by the run where only factors A and B are at their high levels. The next four runs listed consist of the previous runs altered so that factor C is at its high level in each respective run. Similarly, the next eight runs listed consist of the previous runs altered so that factor D is at its high level in each respective run. The process is continued until all 2^k treatment combinations have been listed. [4; 84, 91, 103-104] Note that the observations are taken in random order in the experiment. [1; 1237], [5; 323] Consequently, the run order of the experiment is not necessarily standard order.

The I column in Table 1 contains a plus sign in every row and, as a consequence of the fact that the runs are listed in standard order, the column for the ith average factor effect ($i = 1,...,4$) contains alternating sets of 2^{i-1} minus and plus signs. We can think of the minus and plus signs as shorthand notation for respective coefficients of "–1" and "+1". Note that the coefficients in any interaction effect column can be obtained from those in the factor effect columns for the factors involved. Specifically, each interaction effect coefficient results from the multiplication of the coefficients in the same row of the associated factor effect columns. For instance, each ABC interaction effect column coefficient results from the multiplication of the coefficients in the same row of the A, B, and C factor effect columns.

A table of algebraic signs similar in fashion to Table 1 can be constructed for any 2^k factorial design. First, list the runs of the design in standard order and place a plus sign in every row of the I column. Next, place in the column for the ith average factor effect ($i = 1,2,...,k$) alternating sets of 2^{i-1} minus and plus signs. The table is completed by determining the signs in the interaction effect columns using the type of multiplication described in the previous paragraph.[4; 84, 95, 103-104], [2; 111-112], [5; 322-323]

Table 1 indicates how the numerators of the factorial effects in any 2^4 factorial design are calculated. A given factorial effect numerator is obtained by placing the sign in each row of its column in front of the entry in the same row of the "Run" column. The resulting linear combination of run replicate sums is the numerator of that factorial effect. Note that the numerators of the average effect of factor A and the AB and ABC interaction effects obtained in this way from Table 1 are identical to those shown in equations 1, 2, and 3, respectively. The factorial effect numerators in any 2^k factorial design are determined analogously from its corresponding table of algebraic signs.[4; 84, 95, 103-104], [5; 322-323]

The three-factor interaction effect involving any three factors is defined as one half of the difference between the two-factor interaction effects involving the first two when the third is at its respective high and low levels.[4; 95, 103-104], [5; 315-316]* For example, the ABC interaction effect in a 2^4 factorial design is defined as follows:

$$ABC = \frac{1}{2}[\frac{1}{2}\left(\left[\frac{abc+abcd}{2n}-\frac{bc+bcd}{2n}\right]-\left[\frac{ac+acd}{2n}-\frac{c+cd}{2n}\right]\right)-$$

$$\frac{1}{2}\left(\left[\frac{ab+abd}{2n}-\frac{b+bd}{2n}\right]-\left[\frac{a+ad}{2n}-\frac{(1)+d}{2n}\right]\right)]=$$

$$\frac{1}{8n}\left[-(1)+a+b-ab+c-ac-bc+abc-d+ad+bd-abd+cd-acd-bcd+abcd\right] \quad (3)$$

Higher-order interaction effects for appropriately sized 2^k factorial designs are defined analogously.[4; 103]

Table 1. Algebraic Signs for Calculating Factorial Effects in the 2^4 Factorial Design

	Identity and Factorial Effect Columns															
Run	I	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD
(1)	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+
a	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
b	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-
ab	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
c	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-
ac	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
bc	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
abc	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
d	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-
ad	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
bd	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+
abd	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
cd	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+
acd	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
bcd	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
abcd	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

* See the final paragraph in **1. Introduction.**

Two important observations can be made regarding the parameters of the factor effects model for a 2^k factorial design. First, there are two main effects for any given factor and these have the same magnitude, but opposite signs. Secondly, the number of h-factor true interaction effects ($h = 2,3,...,k$) for a given set of h factors is 2^h. Furthermore, for a given collection of 2^h such true interaction effects, all have the same magnitude, but half will be positive and half will be negative.[1; 1234-1236] These results follow directly from the constraints applied to the main effects and true interaction effects and the fact that each factor has two levels.

The following relationships exist between the factorial effects from a 2^k factorial design as defined in this paper and the least squares estimates of the main effects and true interaction effects from the associated factor effects model. The average effect of a given factor is two times the least squares estimate of the main effect corresponding to the high level of that factor. Similarly, the h-factor interaction effect involving a subset of h factors ($h \le k$) is two times the least squares estimate of the true h-factor interaction effect corresponding to the high levels of all h factors.[4; 86-87]*, [1; 1239]* Furthermore, the least squares estimates of the main effects and true interaction effects are unbiased.[4; 24] Consequently, the factorial effects as defined in this paper are unbiased estimates of two times the appropriate main effect or true interaction effect. These results follow because the factor effects model can be expressed as the multiple linear regression model described in [1; 1234-1236].

When the factor effects model ANOVA is conducted for an unreplicated 2^k factorial design, it is not possible to calculate the MSE if the model contains all $2^k - 1$ main effect and true interaction effect terms. This is because the sum of squares corresponding to each model term has one associated degree of freedom while only $2^k - 1$ degrees of freedom correspond to the total sum of squares (SST). Thus, no degrees of freedom remain to calculate the MSE.
[1; 685, 826, 936, 924, 1241, 875, 815-816, 806]

Furthermore, the least squares fitted value for any given run is the sample average of all the replicates of that run.
[1; 678-679, 672-673, 693-694, 819-820, 814-816, 806, 933-934, 931-932, 927, 924]
Consequently, each observation in an unreplicated 2^k factorial design is also its corresponding least squares fitted value. This, in turn, gives each observation a zero residual which causes the error sum of squares (SSE) to be zero.
[1; 680, 682, 672-673, 819-820, 814-816, 806, 875, 821, 933-934, 931-932, 927, 924]

* See the final paragraph in **1. Introduction.**

The numerator of the mean of all the observed responses from the 2^k factorial experiment is obtained from the I column using the method described in the previous paragraph.[4; 84, 103] Throughout this paper, the term "factorial effect" will refer to average factor effects and interaction effects. It will not, however, refer to the mean of all the observed responses from the experiment.

The product of each coefficient in the I column with any other coefficient in the same row yields the other coefficient back again. Consequently, the set of coefficients in the I column is an identity element in the row-wise multiplication of coefficients among columns. [4; 96, 103] Furthermore, the I column is referred to as the *identity column*.[4; 135, 150]*

The row-wise multiplication of coefficients in any set of columns in Table 1 gives the respective coefficients in one of the columns of Table 1. For example, the products of the coefficients in the BD, ABC, and $ABCD$ interaction effect columns are the respective coefficients in the B effect column. The resulting factorial effect column can be determined by combining the letters designating each column involved, eliminating letters that have even exponents (e.g. A^2, A^4, A^6,...), and reducing any odd exponents to a "1". We now show the use of this procedure for the example just given.

$$(BD)(ABC)(ABCD) = A^2B^3C^2D^2 = B^1 = B. \quad (4)$$

Analogous results apply to the table of algebraic signs for any 2^k factorial design. [4; 95-96, 103-104] The rules for simplifying exponents follow from the fact that the identity column results when each coefficient in a given average factor effect column is squared.

The definitions of the *factor effects model* as well as the associated parameter constraints for 2^k factorial designs involving one, two, and three factors are given in the reference text by Neter, Kutner, Nachtsheim, and Wasserman. Such definitions and constraints for 2^k factorial designs involving more factors follow analogously. It is assumed in these models that the error terms are independent, normally distributed random variables with true mean, zero, and constant variance, σ^2.[1; 693-694, 671-673, 815-816, 806, 812, 931-932, 927, 924] *Throughout this paper, we use the terms "main effect" and "true interaction effect" only to denote main effects or interaction effects as they are defined in the appropriate factor effects model.*

* See the final paragraph in **1. Introduction.**

The vector of observations from any 2^k factorial design follows the multivariate normal distribution as defined in the reference text by Sen and Srivastava.[3; 288-289] Note that this definition makes use of Theorem (5.46) regarding equation (5.43) in [1; 197]. Because the vector of the least squares estimated regression coefficients is the product of a matrix of constants and the vector of observations[1; 1238, 226-227], it also has a multivariate normal distribution. This follows from Lemma B.1 in [3; 289].

Furthermore, the covariance between any pair of estimated regression coefficients is zero.[1; 1239, 231, 226-227] We now have that the estimated regression coefficients are mutually independent. This follows from the relationship between covariance and independence for multivariate normal distributions. Specifically, if all pairs of random variables have zero covariance in a random vector that follows such a distribution, then the random variables in the vector are mutually independent.[3; 289] Consequently, the factorial effects are mutually independent because each is twice its corresponding estimated regression coefficient.

The preceding results imply that if all the main effects and true interaction effects from the factor effects model are zero, then the factorial effects are mutually independent, normally distributed random variables with true mean, zero, and constant variance, σ^2_{effect}. In this case, the estimated expected value of the ith ordered factorial effect from any 2^k factorial design is given by

$$z\left(\frac{i-.375}{2^k-1+.25}\right)\sigma_{effect}. \quad (6)$$

Note that $z(A)$ represents the $A(100)$th percentile of the $N(0,1)$ distribution.[3; 101-102]

The pages cited from reference textbook 1 in the following two paragraphs actually contain discussion of normal probability plots of the least squares estimates of the main effects and true interaction effects that correspond to the factorial effects. However, because of the functional relationship between these two sets of estimators, normal probability plots of either set are interpreted in the same way.

One solution to the problem addressed in the previous two paragraphs is applicable if it is probable that some of the true higher-order interaction effects are zero or quite small in magnitude. In such cases, we can drop those true interaction effect terms from the model and use as the SSE the total of their associated sums of squares. The combined degrees of freedom corresponding to those sums of squares can then be used as the error degrees of freedom. [1; 875-876, 815-816, 806, 824, 837, 937-938, 924], [4; 104] However, the use of this procedure is incorrect if any of those true higher-order interaction effects that are thought to be unimportant actually have relatively large magnitudes.[4; 104] A discussion of the consequences of such an error is given in [1; 882, 876-881, 924].

Terms can also be dropped from a given factor effects model when it is possible to calculate the MSE for that model. The error sum of squares for the updated model is the total of the initial error sum of squares and the sums of squares associated with the terms to be dropped. Similarly, the error degrees of freedom for the updated model is the sum of the initial error degrees of freedom and the degrees of freedom corresponding to the terms to be dropped. [1; 837, 815-816, 806, 937, 924]

A second solution to the lack-of-the-MSE problem is the use of a normal probability plot of the factorial effects. The following paragraphs discuss the underlying theory, construction, and interpretation of these plots.

The following statements regarding the distributions of factorial effects can be made if the factor effects model assumptions have been satisfied. First, each factorial effect from any 2^k factorial design is normally distributed with a mean equal to twice its corresponding main effect or true interaction effect. Secondly, the variance of each factorial effect from any 2^k factorial design is

$$\sigma^2_{effect} = \frac{4\sigma^2}{2^k n} \quad (5)$$

where σ^2 is the error variance of the observations and n is the number of run replicates. This follows from the use of result (A.31) in [1; 1318] after noting that the errors for the observations are mutually independent.

The proof that the factorial effects from any 2^k factorial design are mutually independent will be outlined in this and subsequent paragraphs. First note that the factor effects model can be expressed as the multiple linear regression model discussed in the reference text by Neter, Kutner, Nachtsheim, and Wasserman. The true non-intercept coefficients in this regression model are the main effects and true interaction effects that correspond to the factorial effects. [1; 1234-1236]

The notion that the majority of higher-order true interaction effects are inconsequential is part of the *Sparsity-of-Effects Principle.* This principle states that for any process, usually certain main effects and lower-order true interaction effects comprise the bulk of the effect parameters that are relatively large in magnitude.[4; 104, 134] When the number of candidate factors is somewhat large, it is not unusual to expect that three-factor and higher-order true interaction effects are at most quite small in magnitude.[4; 157] We therefore can assume that those true interaction effects are zero in the corresponding factor effects model. A second assumption made when analyzing fractional factorial designs is that no two-factor true interaction effect will be relatively large in magnitude unless the main effects for at least one of the factors involved are also relatively large in magnitude.

Fractional factorial designs result from selectively removing runs from an unreplicated 2^k factorial design. For example, refer to the 2^4 factorial design shown in Table 1. We may decide to perform only the runs in the rows where the entry in the $ABCD$ interaction effect column is a plus sign. Table 2 shows the fractional factorial design consisting of the runs meeting the criterion and Table 3 shows the fractional factorial design consisting of the remaining runs. The fractional factorial designs shown in Tables 2 and 3 are given in "**Table 4.11 Selected 2^{k-p} Fractional Factorial Designs**" in the reference text by Myers and Montgomery.[4; 158-159]

Table 2. The 2_{IV}^{4-1} Design Having $ABCD$ As Its Generator

	Identity and Aliased Factorial Effect Columns															
Run	I	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD
(1)	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+
ad	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
bd	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+
ab	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
cd	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+
ac	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
bc	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
abcd	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 3. The 2_{IV}^{4-1} Design Having $-ABCD$ As Its Generator

	Identity and Aliased Factorial Effect Columns															
Run	I	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD
d	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-
a	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
b	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-
abd	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
c	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-
acd	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
bcd	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
abc	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-

The i th ordered factorial effect is paired with the standard normal percentile in its estimated expected value from (6) and the ordered pairs are then plotted. [1; 1246-1247, 856-857, 818, 713-716] Such normal probability plots in this paper have the standard normal percentiles on the vertical axis and the factorial effects on the horizontal axis. The points in the plot should have a relatively linear pattern if the true mean of each factorial effect is actually zero. Furthermore, the points in the center of the plot should have this type of pattern because they usually correspond to factorial effects having true means that are zero or quite small in magnitude. A line is subjectively fit to the points that appear to generally follow the same line as those in the center of the plot. The factorial effects corresponding to points located relatively far from this line are considered to have true means that are potentially relatively large in magnitude.[1; 1246-1247]*, [4; 104-106] We can then conclude that each main effect or true interaction effect component in those true means is also potentially relatively large in magnitude.

If every run has been replicated the same number of times, this analysis procedure may also be used with replicated 2^k factorial designs.[1; 1247] Furthermore, this analysis procedure may be used if some of the higher-order interaction effects are missing because their corresponding true interaction effects have been dropped from the model.

Note that the theory for normal probability plots of factorial effects is based on the factor effects model assumptions. Therefore, these plots should not be used if there is evidence of serious violations of those assumptions.

An unreplicated 2^k factorial design can be projected into a replicated two-level factorial design involving a proper subset of the initial k factors. Suppose, after the original analysis, that we consider all of the main effects and true interaction effects involving h of the k factors ($0 < h < k$) to be zero or quite small in magnitude. Then the data from the initial experiment can be analyzed as if from a 2^l factorial experiment involving the other $l = k - h$ factors. Furthermore, the projected design will have 2^h replicates of each run.[4; 109]

3. 2^{k-p} Fractional Factorial Designs.

Fractional factorial designs are used in *screening experiments* to determine which of numerous candidate factors have a substantial impact on the process of interest when it is assumed that some of the higher-order true interaction effects are zero or quite small in magnitude. Typically, many of the original candidate factors do not have a meaningful impact on the process. The subset of factors that are suggested to be substantively influencing the process can then be analyzed in greater detail with follow-up experimentation.[4; 134]

* See the final paragraph in **1. Introduction.**

The two-factor interaction effects involving pairs of the factors A, B, and C obtained from the fractional factorial design shown in Table 2 are

$$l_{AB} = \frac{1}{2}\left[\left(\frac{ab+abcd}{2}-\frac{bd+bc}{2}\right)-\left(\frac{ad+ac}{2}-\frac{(1)+cd}{2}\right)\right]=$$

$$\frac{1}{4}[(1)-ad-bd+ab+cd-ac-bc+abcd] \quad (18)$$

$$l_{AC} = \frac{1}{2}\left[\left(\frac{ac+abcd}{2}-\frac{cd+bc}{2}\right)-\left(\frac{ad+ab}{2}-\frac{(1)+bd}{2}\right)\right]=$$

$$\frac{1}{4}[(1)-ad+bd-ab-cd+ac-bc+abcd] \quad (19)$$

$$l_{BC} = \frac{1}{2}\left[\left(\frac{bc+abcd}{2}-\frac{cd+ac}{2}\right)-\left(\frac{bd+ab}{2}-\frac{(1)+ad}{2}\right)\right]=$$

$$\frac{1}{4}[(1)+ad-bd-ab-cd-ac+bc+abcd] \quad (20)$$

The three-factor interaction effect involving the factors A, B, and C obtained from the fractional factorial design shown in Table 2 is

$$l_{ABC} = \frac{1}{2}\left[\frac{1}{2}([abcd-bc]-[ac-cd])-\frac{1}{2}([ab-bd]-[ad-(1)])\right]=$$

$$\frac{1}{4}[-(1)+ad+bd-ab+cd-ac-bc+abcd] \quad (21).$$

The following eight pairs of columns in Table 2 are identical:

$I = ABCD$ (7)

$A = BCD$ (8)

$B = ACD$ (9)

$C = ABD$ (10)

$AB = CD$ (11)

$AC = BD$ (12)

$BC = AD$ (13)

$ABC = D.$ (14)

The average effects of factors A, B, and C obtained from the fractional factorial design shown in Table 2 are

$$l_A = \frac{ad + ab + ac + abcd}{4} - \frac{(1) + bd + cd + bc}{4} =$$

$$\frac{1}{4}[-(1) + ad - bd + ab - cd + ac - bc + abcd] \quad (15)$$

$$l_B = \frac{bd + ab + bc + abcd}{4} - \frac{(1) + ad + cd + ac}{4} =$$

$$\frac{1}{4}[-(1) - ad + bd + ab - cd - ac + bc + abcd] \quad (16)$$

$$l_C = \frac{cd + ac + bc + abcd}{4} - \frac{(1) + ad + bd + ab}{4} =$$

$$\frac{1}{4}[-(1) - ad - bd - ab + cd + ac + bc + abcd] \quad (17)$$

$AC + BD =$

$$\frac{1}{8}\left[(1) - a + b - ab - c + ac - bc + abc + d - ad + bd - abd - cd + acd - bcd + abcd\right] +$$

$$\frac{1}{8}\left[(1) + a - b - ab + c + ac - bc - abc - d - ad + bd + abd - cd - acd + bcd + abcd\right] =$$

$$\frac{1}{4}\left[(1) - ad + bd - ab - cd + ac - bc + abcd\right] = l_{AC} \quad (26)$$

$BC + AD =$

$$\frac{1}{8}\left[(1) + a - b - ab - c - ac + bc + abc + d + ad - bd - abd - cd - acd + bcd + abcd\right] +$$

$$\frac{1}{8}\left[(1) - a + b - ab + c - ac + bc - abc - d + ad - bd + abd - cd + acd - bcd + abcd\right] =$$

$$\frac{1}{4}\left[(1) + ad - bd - ab - cd - ac + bc + abcd\right] = l_{BC}. \quad (27)$$

$ABC + D =$

$$\frac{1}{8}\left[-(1) + a + b - ab + c - ac - bc + abc - d + ad + bd - abd + cd - acd - bcd + abcd\right] +$$

$$\frac{1}{8}\left[-(1) - a - b - ab - c - ac - bc - abc + d + ad + bd + abd + cd + acd + bcd + abcd\right] =$$

$$\frac{1}{4}\left[-(1) + ad + bd - ab + cd - ac - bc + abcd\right] = l_{ABC}. \quad (28)$$

Therefore, each of the preceding estimates from the fractional factorial design is the same linear combination of run replicate totals as a certain sum of factorial effects from the 2^4 factorial design. Consequently, each of these fractional factorial estimates serves as an estimate of what its corresponding factorial effect sum would have been had a 2^4 factorial design been performed.

We now establish the following relationships between the factorial effects from the 2^4 factorial design shown in Table 1 and the preceding estimates from the fractional factorial design shown in Table 2:

$$A + BCD = \frac{1}{8}[-(1) + a - b + ab - c + ac - bc + abc - d + ad - bd + abd - cd + acd - bcd + abcd] +$$

$$\frac{1}{8}[-(1) - a + b + ab + c + ac - bc - abc + d + ad - bd - abd - cd - acd + bcd + abcd] =$$

$$\frac{1}{4}[-(1) + ad - bd + ab - cd + ac - bc + abcd] = l_A \quad (22)$$

$$B + ACD = \frac{1}{8}[-(1) - a + b + ab - c - ac + bc + abc - d - ad + bd + abd - cd - acd + bcd + abcd] +$$

$$\frac{1}{8}[-(1) + a - b + ab + c - ac + bc - abc + d - ad + bd - abd - cd + acd - bcd + abcd] =$$

$$\frac{1}{4}[-(1) - ad + bd + ab - cd - ac + bc + abcd] = l_B \quad (23)$$

$$C + ABD = \frac{1}{8}[-(1) - a - b - ab + c + ac + bc + abc - d - ad - bd - abd + cd + acd + bcd + abcd] +$$

$$\frac{1}{8}[-(1) + a + b - ab - c + ac + bc - abc + d - ad - bd + abd + cd - acd - bcd + abcd] =$$

$$\frac{1}{4}[-(1) - ad - bd - ab + cd + ac + bc + abcd] = l_c \quad (24)$$

$$AB + CD = \frac{1}{8}[(1) - a - b + ab + c - ac - bc + abc + d - ad - bd + abd + cd - acd - bcd + abcd] +$$

$$\frac{1}{8}[(1) + a + b + ab - c - ac - bc - abc - d - ad - bd - abd + cd + acd + bcd + abcd] =$$

$$\frac{1}{4}[(1) - ad - bd + ab + cd - ac - bc + abcd] = l_{AB} \quad (25)$$

The *generators* for any fractional factorial design are the factorial effects with a coefficient of "+1" or "-1" that are used in the run selection process. A run is included in the fraction only if it has a coefficient of "+1" in the numerator of each generator. The *defining relation* for a fractional factorial design shows all columns having the same algebraic signs as the identity column in a table of algebraic signs for the design.[5; 383-384, 379, 377, 409, 411]*, [4; 135, 150, 157]*, [2; 149-151]* Therefore, for the fractional factorial designs shown in Tables 2 and 3, $ABCD$ and $-ABCD$ are the respective generators and the respective defining relations are

$$I = ABCD \quad (36)$$

$$I = -ABCD. \quad (37)$$

Factorial effects with a coefficient of "+1" or "-1" in the defining relation for a fractional factorial design are sometimes generically called *words*.[4; 135, 150, 169]*, [5; 409]* For instance, $ABCD$ is the only word in the defining relation in (36) while $-ABCD$ is the only word in the defining relation in (37).

Note that the alias structure for the fractional factorial design shown in Table 2 is also given in equations (8) through (14). Specifically, any factorial effects in the same equation are aliases of each other. Note that equations (8) through (14) and expressions (29) through (35) imply that the defining relation shown in equation (7) and also in equation (36) is the defining relation for the design. The alias structure for any fractional factorial design can be shown analogously.

* See the final paragraph in **1. Introduction.**

In general, each factorial effect calculated from any fractional factorial design serves as an estimate of what a certain linear combination of factorial effects from the associated full factorial would have been. Factorial effects belonging to the same linear combination are referred to as *aliases*.[4; 136-137, 150, 157]*, [2; 149-151]*

The alias structure for the fractional factorial design shown in Table 2 can be expressed as shown below and the alias structure for any fractional factorial design can be expressed analogously. The arrow pointing to the right signifies that the quantity to the left of it serves as an estimate of the quantity to the right of it.[2; 149]*

$$l_A \rightarrow A + BCD \quad (29)$$

$$l_B \rightarrow B + ACD \quad (30)$$

$$l_C \rightarrow C + ABD \quad (31)$$

$$l_{AB} \rightarrow AB + CD \quad (32)$$

$$l_{AC} \rightarrow AC + BD \quad (33)$$

$$l_{BC} \rightarrow BC + AD \quad (34)$$

$$l_{ABC} \rightarrow ABC + D. \quad (35)$$

The alias structure shown in expressions (29) through (35) causes ambiguity in the interpretation of the aliased factorial effects. For example, suppose the aliased factor B effect is relatively large in magnitude. Note from expression (30) that this effect estimates the sum of the B effect and the ACD interaction effect from a full 2^4 factorial design. Consequently, we cannot conclude whether the result is due to large magnitude factor B main effects, true ACD interaction effects, or both. All fractional factorial designs are subject to this kind of ambiguity.

* See the final paragraph in **1. Introduction.**

The fractional factorial design that results from the defining relation in (40) is shown in Table 4 below.

Table 4. The 2_{IV}^{6-2} Fractional Factorial Design Using $BCDE$ and $ACDF$ As Generators

	Aliased Factor Effect Columns					
Run	A	B	C	D	E=BCD	F=ACD
(1)	-	-	-	-	-	-
af	+	-	-	-	-	+
be	-	+	-	-	+	-
abef	+	+	-	-	+	+
cef	-	-	+	-	+	+
ace	+	-	+	-	+	-
bcf	-	+	+	-	-	+
abc	+	+	+	-	-	-
def	-	-	-	+	+	+
ade	+	-	-	+	+	-
bdf	-	+	-	+	-	+
abd	+	+	-	+	-	-
cd	-	-	+	+	-	-
acdf	+	-	+	+	-	+
bcde	-	+	+	+	+	-
abcdef	+	+	+	+	+	+

Any fractional factorial design has p *independent* generators and $2^p - p - 1$ generalized interactions in its defining relation.[4; 157] The p generators are referred to as being independent because none is a product involving a subset of any of the rest.[4; 122] Also, every factorial effect is aliased with $2^p - 1$ other factorial effects. The number of observations in such designs involving k factors is 2^{k-p} and these designs are referred to as 2^{k-p} *fractional factorial designs* or *1 / 2^p fractions* of the associated full factorial design.[4; 157]

The defining relation for any 2^{k-p} fractional factorial design also is a tool to identify its alias structure. The process involves sequentially multiplying each element in the defining relation by each of the factorial effects whose aliases have not yet been determined. The resulting products are simplified by eliminating letters that have even exponents and reducing any odd exponents to a "1".[4; 137, 150, 157]

If we desire to run a fraction of an unreplicated 2^6 factorial design that only contains one quarter of the 64 runs, then we must specify two design generators.[4; 150, 157] Suppose that the $BCDE$ and $ACDF$ interaction effects are chosen as design generators. Therefore, we have that

$$I = BCDE = ACDF \quad (38)$$

in a table of algebraic signs for the design. Consequently, the entries in the

$$BCDE(ACDF) = ABEF \quad (39)$$

column of such a table must also be all plus signs. Therefore, the defining relation for this design is

$$I = BCDE = ACDF = ABEF. \quad (40)$$

A word in the defining relation for a fractional factorial design that is not a design generator is referred to as a *generalized interaction*.[4; 150, 157] Note that $ABEF$ is the generalized interaction in the defining relation given by (40). Generalized interactions are products of design generators that are simplified by eliminating letters that have even exponents and reducing any odd exponents to a "1".[4; 121-122]

The number of letters in the word with the fewest letters in the defining relation for a 2^{k-p} fractional factorial design is also its resolution.[2; 154], [5; 385], [4; 139] Consequently, the ½ fractions of an unreplicated 2^4 factorial design shown in Tables 2 and 3 each have resolution IV. These resolution results agree with those reported in Table 4.11 in the reference text by Myers and Montgomery.[4; 158-159] Note that the 2^{6-2} fractional factorial design shown in Table 4 also has resolution IV.

The observations in a 2^{k-p} fractional factorial design of resolution R can be analyzed as a full factorial experiment in subsets containing $R-1$ of the initial k factors.[4; 160] Because fractional factorial designs can be projected into full factorial designs, all analysis procedures discussed in Section 2 for full factorials also apply to fractional factorials. However, the alias structures of fractional factorial designs cause ambiguity in the interpretation of their aliased factorial effects.

Also, the observations in a 2^{k-p} fractional factorial design can be projected into either a fractional or full factorial design in subsets containing $l \leq k-p$ of the initial k factors. The projection will be a full factorial for such subsets whose factor letters do not constitute the letters of a word in the defining relation for the original fractional factorial design.[4; 160]

Myers and Montgomery caution that in most cases, conclusions other than those drawn from the analysis of the design projections discussed in the previous two paragraphs can be made in terms of some of the higher-order true interaction effects. Consequently, they advise checking the results of such analyses with additional experimentation.[4; 160]

4. Fold-over Designs.

A strategy referred to as *fold-over* uses two resolution III fractional factorial designs to eliminate two-factor interaction effect aliases from some or all of the average factor effects. When comparing the tables of algebraic signs for the two fractions, the average factor effects for which this is to be done have opposite signs in their respective columns. A *full fold-over* is done if the signs in every average factor effect column of the initial fraction are changed in the second fraction.[4; 164-165] The mechanics of the full fold-over procedure will be shown with the example that follows.

Using Table 4.11 from the reference text by Myers and Montgomery, a 2^{6-3} design having resolution III is obtained by using ABD, ACE, and BCF as design generators. [4; 158-159] This fractional factorial design will be used as the first half of the example full fold-over design and its defining relation is

$$I = ABD = ACE = BCF = DEF = ABEF = ACDF = BCDE. \quad (57)$$

This design is shown in Table 5.

The alias structure for the 2^{6-2} fractional factorial design shown in Table 4 is as follows:

$$I = BCDE = ACDF = ABEF \quad (41)$$
$$A = BEF = CDF = ABCDE \quad (42)$$
$$B = AEF = CDE = ABCDF \quad (43)$$
$$C = ADF = BDE = ABCEF \quad (44)$$
$$D = ACF = BCE = ABDEF \quad (45)$$
$$E = ABF = BCD = ACDEF \quad (46)$$
$$F = ABE = ACD = BCDEF \quad (47)$$
$$AB = EF = ACDE = BCDF \quad (48)$$
$$AC = DF = ABDE = BCEF \quad (49)$$
$$AD = CF = ABCE = BDEF \quad (50)$$
$$AE = BF = ABCD = CDEF \quad (51)$$
$$AF = BE = CD = ABCDEF \quad (52)$$
$$BC = DE = ABDF = ACEF \quad (53)$$
$$BD = CE = ABCF = ADEF \quad (54)$$
$$ABC = ADE = BDF = CEF \quad (55)$$
$$ABD = ACE = BCF = DEF. \quad (56)$$

The resolution of a fractional factorial design is a measure of its aliasing properties. If every factorial effect containing q letters is only aliased with factorial effects containing $R-q$ or more letters, then the design has resolution R. Roman numeral subscripts are used to identify the resolution of a fractional factorial design.[5; 385] The practical implications that follow from three such resolutions are shown below.

1. *Resolution III Designs.* Average factor effect pairs are not aliased, but at least one average factor effect is aliased with a two-factor interaction effect. Also, pairs of two-factor interaction effects may be aliased.[5; 385], [4; 138]*

2. *Resolution IV Designs.* Average factor effect pairs are not aliased and average factor effects are not aliased with any two-factor interaction effects. However, at least one pair of two-factor interaction effects is aliased.[5; 385]

3. *Resolution V Designs.* Average factor effects are not aliased with other average factor effects or two-factor interaction effects. Furthermore, all pairs of two-factor interaction effects are unaliased. However, at least one two-factor interaction effect is aliased with a three-factor interaction effect.[5; 385]

* See the final paragraph in **1. Introduction.**

The second half of the example full fold-over design, shown in Table 6, is obtained by reversing the signs in every aliased factor effect column of Table 5. The generators for the second half are $-ABD$, $-ACE$, and $-BCF$, and the defining relation for this half is

$$I = -ABD = -ACE = -BCF = -DEF = ABEF = ACDF = BCDE. \quad (72)$$

Table 6. The 2_{III}^{6-3} Fractional Factorial Design Using $-ABD$, $-ACE$, and $-BCF$ As Generators

	Aliased Factor Effect Columns					
Run	A	B	C	D=-AB	E=-AC	F=-BC
abc	+	+	+	-	-	-
bcde	-	+	+	+	+	-
acdf	+	-	+	+	-	+
cef	-	-	+	-	+	+
abef	+	+	-	-	+	+
bdf	-	+	-	+	-	+
ade	+	-	-	+	+	-
(1)	-	-	-	-	-	-

Furthermore, the alias structure for the 1/8 fraction shown in Table 6 is as follows:

$$l'_A \to A - BD - CE + BEF + CDF - ABCF - ADEF + ABCDE \quad (73)$$

$$l'_B \to B - AD - CF + AEF + CDE - ABCE - BDEF + ABCDF \quad (74)$$

$$l'_C \to C - AE - BF + ADF + BDE - ABCD - CDEF + ABCEF \quad (75)$$

$$l'_D \to D - AB - EF + ACF + BCE - ACDE - BCDF + ABDEF \quad (76)$$

$$l'_E \to E - AC - DF + ABF + BCD - ABDE - BCEF + ACDEF \quad (77)$$

$$l'_F \to F - BC - DE + ABE + ACD - ABDF - ACEF + BCDEF \quad (78)$$

$$l'_{AF} \to AF + BE + CD - ABC - ADE - BDF - CEF + ABCDEF. \quad (79)$$

Table 5. The 2_{III}^{6-3} Fractional Factorial Design Using ABD, ACE, and BCF As Generators

	Aliased Factor Effect Columns					
Run	A	B	C	D=AB	E=AC	F=BC
def	-	-	-	+	+	+
af	+	-	-	-	-	+
be	-	+	-	-	+	-
abd	+	+	-	+	-	-
cd	-	-	+	+	-	-
ace	+	-	+	-	+	-
bcf	-	+	+	-	-	+
abcdef	+	+	+	+	+	+

The alias structure for the 1/8 fraction shown in Table 5 is as follows:

$$l_A \rightarrow A + BD + CE + BEF + CDF + ABCF + ADEF + ABCDE \quad (58)$$
$$l_B \rightarrow B + AD + CF + AEF + CDE + ABCE + BDEF + ABCDF \quad (59)$$
$$l_C \rightarrow C + AE + BF + ADF + BDE + ABCD + CDEF + ABCEF \quad (60)$$
$$l_D \rightarrow D + AB + EF + ACF + BCE + ACDE + BCDF + ABDEF \quad (61)$$
$$l_E \rightarrow E + AC + DF + ABF + BCD + ABDE + BCEF + ACDEF \quad (62)$$
$$l_F \rightarrow F + BC + DE + ABE + ACD + ABDF + ACEF + BCDEF \quad (63)$$
$$l_{AF} \rightarrow AF + BE + CD + ABC + ADE + BDF + CEF + ABCDEF. \quad (64)$$

If we assume that all three-factor and higher-order true interaction effects are zero, the alias structure shown in expressions (58) through (64) can be reduced as follows:

$$l_A \rightarrow A + BD + CE \quad (65)$$
$$l_B \rightarrow B + AD + CF \quad (66)$$
$$l_C \rightarrow C + AE + BF \quad (67)$$
$$l_D \rightarrow D + AB + EF \quad (68)$$
$$l_E \rightarrow E + AC + DF \quad (69)$$
$$l_F \rightarrow F + BC + DE \quad (70)$$
$$l_{AF} \rightarrow AF + BE + CD. \quad (71)$$

Aliased average factor effects from each half can also be combined as shown below to obtain the following two-factor interaction effect sums:

$$\frac{l_A - l'_A}{2} = \frac{A + BD + CE - (A - BD - CE)}{2} = \frac{2(BD + CE)}{2} = BD + CE \quad (93)$$

$$\frac{l_B - l'_B}{2} = \frac{B + AD + CF - (B - AD - CF)}{2} = \frac{2(AD + CF)}{2} = AD + CF \quad (94)$$

$$\frac{l_C - l'_C}{2} = \frac{C + AE + BF - (C - AE - BF)}{2} = \frac{2(AE + BF)}{2} = AE + BF \quad (95)$$

$$\frac{l_D - l'_D}{2} = \frac{D + AB + EF - (D - AB - EF)}{2} = \frac{2(AB + EF)}{2} = AB + EF \quad (96)$$

$$\frac{l_E - l'_E}{2} = \frac{E + AC + DF - (E - AC - DF)}{2} = \frac{2(AC + DF)}{2} = AC + DF \quad (97)$$

$$\frac{l_F - l'_F}{2} = \frac{F + BC + DE - (F - BC - DE)}{2} = \frac{2(BC + DE)}{2} = BC + DE. \quad (98)$$

Note that the respective generators for the two halves only differ in their algebraic signs. In general, the generators for the second half of any full fold-over design will be the same as those for the first half except that some signs will be reversed. Specifically, design generators having odd numbers of letters will have opposite signs between the two halves.[4; 166]

The generators for the fractional factorial design consisting of the runs in both halves of any fold-over design can be obtained from the generators for each half. Of the $L+U$ generators for each half, L pairs of generators will be identical and U pairs of generators having the same letters will have opposite signs between the two halves. Each generator in the L pairs of identical generators will also be a generator for the combined design. The remaining $U-1$ generators for the combined design result from the multiplication of even numbers of the U words having opposite signs between the two halves. These $U-1$ generators must be *independent* in that none can be expressed as a product involving a subset of any of the rest. Furthermore, the U words used in the multiplications must all be from the same defining relation.[4; 169, 122]*

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* See the final paragraph in **1. Introduction.**

If we assume that all three-factor and higher-order true interaction effects are zero, the alias structure shown in expressions (73) through (79) can be reduced as follows:

$$l'_A \rightarrow A - BD - CE \quad (80)$$

$$l'_B \rightarrow B - AD - CF \quad (81)$$

$$l'_C \rightarrow C - AE - BF \quad (82)$$

$$l'_D \rightarrow D - AB - EF \quad (83)$$

$$l'_E \rightarrow E - AC - DF \quad (84)$$

$$l'_F \rightarrow F - BC - DE \quad (85)$$

$$l'_{AF} \rightarrow AF + BE + CD. \quad (86)$$

Therefore, the aliased average factor effects from each half can be combined as follows to obtain average factor effects that are not aliased with two-factor interaction effects:

$$\frac{l_A + l'_A}{2} = \frac{A + BD + CE + A - BD - CE}{2} = \frac{2A}{2} = A \quad (87)$$

$$\frac{l_B + l'_B}{2} = \frac{B + AD + CF + B - AD - CF}{2} = \frac{2B}{2} = B \quad (88)$$

$$\frac{l_C + l'_C}{2} = \frac{C + AE + BF + C - AE - BF}{2} = \frac{2C}{2} = C \quad (89)$$

$$\frac{l_D + l'_D}{2} = \frac{D + AB + EF + D - AB - EF}{2} = \frac{2D}{2} = D \quad (90)$$

$$\frac{l_E + l'_E}{2} = \frac{E + AC + DF + E - AC - DF}{2} = \frac{2E}{2} = E \quad (91)$$

$$\frac{l_F + l'_F}{2} = \frac{F + BC + DE + F - BC - DE}{2} = \frac{2F}{2} = F. \quad (92)$$

However, it may be necessary to run the second half of a fold-over design in order to gain more information. As an example, suppose that in the first half shown in Table 5, only the aliased effects of factors B, C, and E are relatively large in magnitude. Note that the aliases of these effects are respectively given in expressions (66), (67), and (69). The following are three different sets of factorial effects whose corresponding main effects and true interaction effects could plausibly be relatively large in magnitude and thus causing these results:

1. B, C, and E
2. B, C, and AC
3. C, AC, and CF.

Because these and several other explanations are also possible, the second half of the fold-over design must be run in order to reach a conclusion.

5. Full Fold-over Design Examples.

The code shown in the Data Appendix was written for and the corresponding edited output shown there was generated using SAS/STAT® software, Version 8, of the SAS System. Copyright © 1999 SAS Institute Inc. SAS and all other SAS Institute Inc. product or service names are registered trademarks or trademarks of SAS Institute Inc., Cary, NC, USA.[1] This software was run on a UNIX® operating system.[2]

The following problem scenario and the associated data set were obtained from the reference text by Box and Draper. The response variable was the strength of dyestuff made in an industrial setting and higher strength values are considered better. The six factors thought to potentially have meaningful influence over the process and their corresponding alphabetic letter designations are listed below.

Polysulfide Index	Factor A
Reflux Rate	Factor B
Moles of Polysulfide	Factor C
Reaction Time	Factor D
Amount of Solvent	Factor E
Reaction Temperature	Factor F

The factor Time was measured in minutes, the factor Solvent in cubic centimeters, and the factor Temperature in degrees Celsius.[2; 114-116]

[1] SAS and all other SAS Institute Inc. product or service names are registered trademarks or trademarks of SAS Institute Inc. in the USA and other countries. ® indicates USA registration.

[2] UNIX® is a registered trademark of The Open Group in the United States and other countries.

The procedure discussed in the previous paragraph will now be used to determine the generators for the fractional factorial design consisting of the runs in Tables 5 and 6. Recall that the designs shown in those two tables were the respective first and second halves of the full fold-over design example. The generators for the first half were ABD, ACE, and BCF while those for the second half were $-ABD$, $-ACE$, and $-BCF$. There were no identical generators between the two sets so $L=0$. We now will determine the remaining $U-1=3-1=2$ words. The generators for the first half will be used in the computations.

$$ABD(ACE)=A^2BCDE=BCDE \quad (99)$$
$$ABD(BCF)=AB^2CDF=ACDF. \quad (100)$$

Note that the runs in the combined halves of the full fold-over design are the same as those in the 2_{IV}^{6-2} fractional factorial design shown in Table 4 because the same generators correspond to each design. Furthermore, the corresponding effects from each design involve the same linear combinations of runs. The difference between the two designs is that the combined full fold-over design is run in two blocks where each half constitutes one block. No blocking is used in the design shown in Table 4.

In general, the runs in the combined halves of any fold-over design will be the same as those in a certain fractional factorial design. The only difference between the two designs is that each half constitutes one block in the combined design while no blocking is used in the other design.

In any combined fold-over design, it is assumed that the variance of the observations is the same within and between the blocks and that there is no interaction between the treatments and blocks. The confounding scheme associated with the type of blocking used in fold-over procedures will not be discussed in this paper. However, blocking in this fashion can result in a reduction of the variance of the factorial effects over that associated with performing the fraction consisting of the runs in the combined design without blocking. Note that all blocks discussed in this paper are fixed.

The assumptions made when analyzing fractional factorial designs might make it so that only the first half of a fold-over design needs to be run. For example, suppose that in the first half shown in Table 5, only the aliased effects of factors C and D and the aliased CD interaction effect are relatively large in magnitude. We may therefore conclude that the main effects of factors C and D and the true CD interaction effects are relatively large in magnitude. This follows because none of the aliases of the C and D effects or the CD interaction effect, shown in expressions (67), (68), and (71), respectively, involve factors C or D.

The data from the 2^6 factorial experiment was then projected into a 2^3 factorial design involving the factors Polysulfide Index, Time, and Temperature. It was assumed during the data analysis that all true interaction effects are zero. The analysis included construction of diagnostic residual plots to check for outliers and serious violations of the factor effects model assumptions. The SAS code used to perform this analysis and the corresponding output are respectively shown on pp. 5-7 and pp. 8-18 of the Data Appendix.

The ordinary and studentized deleted residuals for the projection design are listed on pp. 8-9. The columns containing these residuals are labeled "Resid" and "RSTUDENT", respectively. Neither the plot of the studentized deleted residuals against the fitted values shown on p. 10 nor the normal probability plot of the studentized deleted residuals shown on p. 16 suggests that any of the observations are outliers.

The plot of the ordinary residuals against the fitted values is shown on p. 11 and the plots of the ordinary residuals against the factor levels are shown on pp. 12-14. These plots as well as the plot of the studentized deleted residuals against the fitted values do not suggest any substantial violations of the assumption of constant error variance in the factor effects model.

The plot of the ordinary residuals against the ordering of the runs is shown on p. 15. The plot does not suggest serious problems with the assumption of independent error terms in the factor effects model.

The normal probability plot of the ordinary residuals is shown on p. 17. Neither this plot nor the normal probability plot of the studentized deleted residuals suggests that the assumption of normally distributed error terms is inappropriate in the factor effects model.

The average factor effects and the observed F statistics and p-values from the corresponding ANOVA F tests are shown on p. 18. Note that we again can conclude that the main effects for Polysulfide Index, Time, and Temperature are nonzero using the .5% significance level for each individual hypothesis test. The p-values from those ANOVA F tests are .0005, <.0001, and <.0001, respectively. The R^2 value of .585475 suggests a somewhat large degree of experimental error variance for this process, as was expected.

We therefore have that the diagnostic plots as well as the analysis of variance for the projection design support our earlier conclusion. We now discuss two simulated full fold-over experiments using selected observations from the 2^6 factorial experiment.

Each of the following observations could be made prior to data collection based on the process history. First, the process appears to have a somewhat large degree of experimental error variability. Secondly, the possibly substantive main effects and true interaction effects would probably not be large in magnitude relative to this variability.[2; 116] This suggests that the ANOVA F tests for main effects and true interaction effects might be conservative.

The observations from the unreplicated 2^6 factorial experiment involving the previously listed factors are shown in standard order in "**TABLE 4.4. A 2^6 factorial design and resulting observations of strength, hue, and brightness**" in the reference text by Box and Draper. [2; 114-115] The observed strength values are also shown in the SAS code on p. 1 of the Data Appendix and are in standard order there if one reads from left to right starting with the top row.

The history of the process prior to this experiment as well as other experimental results did not provide conclusive evidence of substantive true interaction effects.[2; 116] We therefore have assumed in our analysis of the 2^6 factorial experiment that all three-factor and higher-order true interaction effects are zero.

The SAS code used to perform the analysis of the 2^6 factorial experiment and the corresponding output are respectively shown on p. 1 and pp. 2-3 of the Data Appendix. The factorial effects are shown on p. 3 and the observed F statistics and p-values from the ANOVA F tests for the corresponding main effects and true interaction effects are shown on p. 2. Note that only the main effects of Polysulfide Index, Time, and Temperature can be concluded to be nonzero at the .5% significance level for each individual hypothesis test. The p-values from those ANOVA F tests are .0007, <.0001, and <.0001, respectively. If the .5% significance level is used for all 21 ANOVA F tests, then an upper bound for the family significance level as given by the *Kimball inequality*[1; 1240, 831, 937, 924] is approximately 10%.

The normal probability plot of the average factor effects and two-factor interaction effects is shown on p. 4 of the Data Appendix along with the SAS code that generates it. The plot suggests that only the main effects of Polysulfide Index, Time, and Temperature are potentially relatively large in magnitude.

It should also be pointed out that we would conclude that the Polysulfide Index X Reflux Rate, Reflux Rate X Time, and Polysulfide Index X Temperature true interaction effects are nonzero if the 10% significance level were used for each individual hypothesis test. The p-values from those ANOVA F tests are .0696, .0796, and .0931, respectively. However, recall that there was not conclusive evidence of substantive true interaction effects from the process history or other experimental results. Based on that lack of evidence and the results discussed in the previous two paragraphs, we conclude, as did the authors, that only the Polysulfide Index, Time, and Temperature main effects are relatively large in magnitude. It is important to note that the authors arrived at their conclusion without assuming that any main effects or true interaction effects are zero.[2; 116-119]

The SAS code used to perform the analysis of the second half of Fold-over Design 1 and its corresponding output are respectively shown on p. 25 and p. 26 of the Data Appendix. The aliased factorial effects that are relatively large in magnitude are listed below.

Effect Type	Aliased Effect
Polysulfide Index	4.1500
Reflux Rate	1.6500
Time	2.6000
Temperature	3.6500

The normal probability plot of the aliased factorial effects from the second half of Fold-over Design 1 is shown on p. 27 of the Data Appendix along with the SAS code that generates it. Note that we again cannot conclude that any linear combinations of main effects and true interaction effects are potentially relatively large in magnitude.

The SAS code used to perform the analysis of the combined halves of Fold-over Design 1 and its corresponding output are respectively shown on p. 28 and p. 29 of the Data Appendix. Each half of the design was treated as a separate block and a Block main effect ANOVA F test was performed. Table 7 shows the effects having relatively large magnitudes along with the observed F statistics and p-values from the corresponding ANOVA F tests.

Table 7. The Effects with Relatively Large Magnitudes and the Corresponding ANOVA F Test Results from the Analysis of the Combined Halves of Fold-over Design 1

Effect Type	Effect	F Statistic	P-value
Polysulfide Index	3.1875	12.20	.1775
Reflux Rate	2.2625	6.15	.2441
Time	3.6875	16.33	.1544
Temperature	2.9625	10.54	.1902

Despite the fact that the factor effects in Table 7 are large relative to the other effects, we cannot conclude that any of the associated main effects are nonzero using the 10% significance level for each individual hypothesis test. The reference distribution used to calculate each p-value is the F(1, 1) distribution and the critical F value for this distribution is 39.86 when the 10% significance level is used. However, the critical F value drops to 8.53 for the F(1,2) distribution when the same significance level is used.[2; 611] Consequently, the F tests are conservative because of the single error degree of freedom available. Furthermore, recall our conjecture at the beginning of this section that the ANOVA F tests for main effects and true interaction effects might be conservative because of process characteristics. Consequently, one could conclude that the main effects corresponding to the average factor effects in Table 7 are potentially nonzero.

The generators for the first half of Fold-over Design 1 are ABD, ACE, and BCF which correspond, respectively, to the following three-factor interaction effects:

Polysulfide Index X Reflux Rate X Time
Polysulfide Index X Moles Polysulfide X Solvent
Reflux Rate X Moles Polysulfide X Temperature.

The runs for this half are listed in Table 5 and, as was discussed in Section 4, this design has resolution III.

The SAS code that generates all 1/8 fractions of a 2^6 factorial design using $\pm ABD$, $\pm ACE$, and $\pm BCF$ as generators is shown on p. 19 of the Data Appendix. The corresponding output showing the 1/8 fractions is on pp. 20-21 of the Data Appendix. The fraction denoted on p. 20 as "FRACTION=1" is the first half of Fold-over Design 1. Note that the order of the runs in this fraction in the SAS output differs from that in Table 5.

The SAS code used to perform the analysis of the first half of Fold-over Design 1 and its corresponding output are respectively shown on p. 22 and p. 23 of the Data Appendix. The aliased factorial effects that are relatively large in magnitude are listed below.

Effect Type	Aliased Effect
Polysulfide Index	2.2250
Reflux Rate	2.8750
Time	4.7750
Temperature	2.2750

The normal probability plot of the aliased factorial effects from the first half of Fold-over Design 1 is shown on p. 24 of the Data Appendix along with the SAS code that generates it. The plot does not suggest that any linear combinations of main effects and true interaction effects are potentially relatively large in magnitude.

Because the analysis of the first half was inconclusive, the experimenters would run the second half in order to gain more information. The second half of Fold-over Design 1 is obtained using $-ABD$, $-ACE$, and $-BCF$ as design generators. This fraction is denoted as "FRACTION=8" on p. 21 of the Data Appendix and is also shown in Table 6. The order of the runs in this fraction in the SAS output differs from that in Table 6.

A comparison of p-values between the analysis of the blocked 2^4 factorial design and that of the combined halves of Fold-over Design 1 shows that the F tests from the latter design are much more conservative. Note that we can conclude from the analysis of the blocked 2^4 factorial design that the Polysulfide Index, Reflux Rate, Time, and Temperature main effects are nonzero using the 1% significance level for each individual hypothesis test.

However, there is not enough evidence to suggest that the Polysulfide Index X Reflux Rate or the Reflux Rate X Time true interaction effects are nonzero using the 10% significance level for each individual hypothesis test. The p-values from those F tests are .1447 and .1905, respectively.

The conclusions regarding the Block main effects are contradictory between the two analyses. We do not have enough evidence to conclude that there are nonzero Block main effects using the 10% significance level in the analysis of the combined halves of Fold-over Design 1. However, we do have enough evidence to make such a conclusion at the 3% significance level in the analysis of the blocked 2^4 factorial design. The conclusion of nonzero Block main effects in the analysis of the blocked 2^4 factorial design may have been caused, at least in part, by the relatively large degree of experimental error variability that appears to be associated with the process.

Recall that our conclusion from the analysis of the 2^6 factorial experiment was that only the Polysulfide Index, Time, and Temperature main effects are relatively large in magnitude. Thus, we consider the conclusion reached from the analysis of the combined halves of Fold-over Design 1 and the subsequent data projection regarding the Reflux Rate main effects to be incorrect. This erroneous result may be at least partially explained by the apparent large degree of experimental error variability in the process. However, we suspect that the analysis of an unreplicated 2^4 factorial design involving the factors Polysulfide Index, Reflux Rate, Time, and Temperature using new observations would suggest that the Reflux Rate factor is unimportant. The correct conclusion would then presumably be reached using a total of 32 experimental runs — 16 for both halves of Fold-over Design 1 and 16 for the follow-up 2^4 factorial design. Note that this is half the number of runs used in the 2^6 factorial experiment. Furthermore, the fold-over procedure has the benefit of incorporating blocking into its design.

The normal probability plot of the average factor effects, the aliased two-factor interaction effects, and the Block effect from the combined halves of Fold-over Design 1 is shown on p. 30 of the Data Appendix along with the SAS code that generates it. The plot suggests that only the Polysulfide Index, Reflux Rate, Time, and Temperature main effects are potentially relatively large in magnitude. We therefore conclude, based on these results and those of the ANOVA F tests, that the main effects of these four factors are the only sizeable effect parameters from the factor effects model.

The p-value of .2900 from the Block main effect F test is well above the 10% significance level. Consequently, using this result and that of the normal probability plot, there is not enough evidence to suggest a lack of homogeneity of experimental conditions between the blocks.

The preceding conclusions were then checked by projecting the 16 observations into a blocked 2^4 factorial design involving the factors Polysulfide Index, Reflux Rate, Time, and Temperature. We again treated each half as a separate block and performed a Block main effect F test. The Polysulfide Index X Reflux Rate and Reflux Rate X Time true interaction effects were the only true interaction effects not assumed to be zero in the corresponding factor effects model. Recall that we would have concluded that both sets of true interaction effects are nonzero if the 10% significance level for each hypothesis test were used in the analysis of the 2^6 factorial experiment. Because of this and the fact that both sets of true interaction effects involve the factor Reflux Rate, we were interested in checking if, in addition to the Reflux Rate main effects, the data also suggested that these true interaction effects are nonzero in the projection of the combined design.

The SAS code used to perform this analysis and its corresponding output are respectively shown on p. 31 and p. 32 of the Data Appendix. Table 8 shows the effects having relatively large magnitudes along with the observed F statistics and p-values from the corresponding ANOVA F tests.

Table 8. The Effects with Relatively Large Magnitudes and the Corresponding ANOVA F Test Results from the Analysis of the Projection of the Combined Halves of Fold-over Design 1 into a Blocked 2^4 Factorial Design

Effect Type	Effect	F Statistic	P-value
Block	-1.8625	7.66	.0244
Polysulfide Index	3.1875	22.44	.0015
Reflux Rate	2.2625	11.30	.0099
Time	3.6875	30.03	.0006
Temperature	2.9625	19.38	.0023

The normal probability plot of the aliased factorial effects from the first half of Fold-over Design 2 is shown on p. 38 of the Data Appendix along with the SAS code that generates it. The plot suggests that the linear combination of effect parameters involving the appropriate Temperature main effect is potentially relatively large in magnitude. Note from expression (107) that none of the two-factor interaction effects aliased with the Temperature effect involves the factor Temperature. Therefore, we can conclude that the Temperature main effects are potentially relatively large in magnitude.

The generators for the second half of Fold-over Design 2 are $-BCD$, $-ABE$, and $-ACF$ and its defining relation is

$$I = -BCD = -ABE = -ACF = -DEF = ABDF = ACDE = BCEF. \quad (109)$$

This half also has resolution III and is denoted as "FRACTION=8" in the SAS output on p. 35 of the Data Appendix.

The alias structure for the second half of Fold-over Design 2 is as follows when three-factor and higher-order true interaction effects are assumed to be zero:

$$l'_A \rightarrow A - BE - CF \quad (110)$$

$$l'_B \rightarrow B - AE - CD \quad (111)$$

$$l'_C \rightarrow C - AF - BD \quad (112)$$

$$l'_D \rightarrow D - BC - EF \quad (113)$$

$$l'_E \rightarrow E - AB - DF \quad (114)$$

$$l'_F \rightarrow F - AC - DE \quad (115)$$

$$l'_{AD} \rightarrow AD + BF + CE. \quad (116)$$

The SAS code used to perform the analysis of the second half of Fold-over Design 2 and its corresponding output are respectively shown on p. 39 and p. 40 of the Data Appendix. The aliased factorial effects that are relatively large in magnitude are listed below.

Effect Type	Aliased Effect
Polysulfide Index	4.5000
Time	4.0000
Temperature	4.9000

The generators for the first half of Fold-over Design 2 are BCD, ABE, and ACF which correspond, respectively, to the following three-factor interaction effects:

Reflux Rate X Moles Polysulfide X Time
Polysulfide Index X Reflux Rate X Solvent
Polysulfide Index X Moles Polysulfide X Temperature.

The defining relation for this half is

$$I = BCD = ABE = ACF = DEF = ABDF = ACDE = BCEF. \quad (101)$$

Note that this half has resolution III because the words with the fewest letters in its defining relation contain three letters.

The alias structure for the first half of Fold-over Design 2 is as follows when three-factor and higher-order true interaction effects are assumed to be zero:

$$l_A \rightarrow A + BE + CF \quad (102)$$
$$l_B \rightarrow B + AE + CD \quad (103)$$
$$l_C \rightarrow C + AF + BD \quad (104)$$
$$l_D \rightarrow D + BC + EF \quad (105)$$
$$l_E \rightarrow E + AB + DF \quad (106)$$
$$l_F \rightarrow F + AC + DE \quad (107)$$
$$l_{AD} \rightarrow AD + BF + CE. \quad (108)$$

The SAS code that generates all 1/8 fractions of a 2^6 factorial design using $\pm BCD$, $\pm ABE$, and $\pm ACF$ as generators is shown on p. 33 of the Data Appendix. The corresponding output showing the 1/8 fractions is on pp. 34-35 of the Data Appendix. The fraction denoted on p. 34 as "FRACTION=1" is the first half of Fold-over Design 2.

The SAS code used to perform the analysis of the first half of Fold-over Design 2 and its corresponding output are respectively shown on p. 36 and p. 37 of the Data Appendix. The aliased factorial effects that are relatively large in magnitude are listed below.

Effect Type	Aliased Effect
Polysulfide Index	2.0250
Time	1.8750
Temperature	3.7750

The preceding conclusion was then checked by projecting the 16 observations into a blocked 2^3 factorial design involving the factors Polysulfide Index, Time, and Temperature. All true interaction effects were assumed to be zero in the subsequent analysis. The SAS code used to perform this analysis and its corresponding output are respectively shown on p. 45 and p. 46 of the Data Appendix. Table 10 shows the effects having relatively large magnitudes along with the observed F statistics and p-values from the corresponding ANOVA F tests. We can conclude that the Polysulfide Index, Time, and Temperature main effects are nonzero using the .5% significance level for each individual hypothesis test.

Table 10. The Effects with Relatively Large Magnitudes and the Corresponding ANOVA F Test Results from the Analysis of the Projection of the Combined Halves of Fold-over Design 2 into a Blocked 2^3 Factorial Design

Effect Type	Effect	F Statistic	P-value
Polysulfide Index	3.2625	15.26	.0024
Time	2.9375	12.37	.0048
Temperature	4.3375	26.97	.0003

The p-value of .9417 from the Block main effect F test is well above the 10% significance level. Consequently, there is not enough evidence to suggest a lack of homogeneity of experimental conditions between the blocks.

We have reached the same conclusion through the analysis of the combined halves of Fold-over Design 2 and the subsequent data projection as was reached from the analysis of the 2^6 factorial experiment using four times fewer observations. Also, the fold-over procedure incorporated blocking into its design.

The usefulness of the full fold-over technique has been shown for a process that appears to satisfy the factor effects model assumptions quite well. The robustness of fold-over procedures when applied to processes with experimental errors that depart from those assumptions will not be covered in this paper.

The normal probability plot of the aliased factorial effects from the second half of Fold-over Design 2 is shown on p. 41 of the Data Appendix along with the SAS code that generates it. The plot does not suggest that any linear combinations of main effects and true interaction effects are potentially relatively large in magnitude.

The defining relation for the fractional factorial design consisting of the runs in the combined halves of Fold-over design 2 is

$$I = ABDF = ACDE = BCEF. \quad (117)$$

Note from this defining relation that the combined design has resolution IV.

The SAS code used to perform the analysis of the combined halves of Fold-over Design 2 and its corresponding output are respectively shown on p. 42 and p. 43 of the Data Appendix. Table 9 shows the effects having relatively large magnitudes along with the observed F statistics and p-values from the corresponding ANOVA F tests.

Table 9. The Effects with Relatively Large Magnitudes and the Corresponding ANOVA F Test Results from the Analysis of the Combined Halves of Fold-over Design 2

Effect Type	Effect	F Statistic	P-value
Polysulfide Index	3.2625	20.97	.1369
Time	2.9375	17.00	.1515
Temperature	4.3375	37.06	.1036

Given the conservative nature of the ANOVA F tests that was described earlier, we can again conclude that the main effects corresponding to the average factor effects in Table 9 are potentially nonzero.

The normal probability plot of the average factor effects, the aliased two-factor interaction effects, and the Block effect from the combined halves of Fold-over Design 2 is shown on p. 44 of the Data Appendix along with the SAS code that generates it. The plot suggests that the Polysulfide Index, Time, and Temperature main effects are potentially relatively large in magnitude. We therefore conclude, based on these results and those of the ANOVA F tests, that the main effects of these three factors are the only sizeable effect parameters from the factor effects model.

The p-value of .9443 from the Block main effect F test is well above the 10% significance level. Consequently, using this result and that of the normal probability plot, there is not enough evidence to suggest a lack of homogeneity of experimental conditions between the blocks.

Bibliography

1. Neter, John; Kutner, Michael H.; Nachtsheim, Christopher J.; Wasserman, William, *Applied Linear Statistical Models, 4th ed.*, RICHARD D. IRWIN, INC., 1989, and Times Mirror Higher Education Group, Inc., 1996.
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The variability of the aliased factorial effects from each half of any fold-over design is higher than the variability of the factorial effects from the associated full factorial design. Also, if the associated Block main effects are sufficiently small, the variability of the effects from the combined design is also higher than the variability of the factorial effects from the corresponding full factorial design. This can be seen from equation (5) when it is noted that any 2^{k-p} fractional factorial design is a full factorial in certain proper subsets of the initial k factors. Consequently, the conclusions reached from the analysis of a fold-over design are usually more likely to be erroneous than are those reached from the analysis of the corresponding full factorial design. Furthermore, this problem is made worse when the substantive main effects and true interaction effects are not large in magnitude relative to the size of the experimental error variability. It is worth mentioning that the simulated full fold-over experiments were effective for this industrial process even though it appears to present the above challenge.

6. Conclusion.

This paper has given a brief introduction to two-level factorial and fractional factorial designs followed by a discussion of fold-over designs. Furthermore, the full fold-over procedure and its utility have been illustrated with two examples. The potential for reducing the number of runs required in screening experiments and the convenience of running fractional factorial designs in blocks are the primary benefits of the fold-over technique.

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The 2^6 Factorial Experiment Analyzed Assuming Three-factor And Higher-order True Interaction Effects Are Zero.

The GLM Procedure

Dependent Variable: Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	21	370.9107812	17.6624182	4.83	<.0001
Error	42	153.7240625	3.6600967		
Corrected Total	63	524.6348438			

R-Square	Coeff Var	Root MSE	Strength Mean
0.706988	17.19916	1.913138	11.12344

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Plyslf_i	1	48.8251562	48.8251562	13.34	0.0007
Reflux	1	7.9101563	7.9101563	2.16	0.1490
Plyslf_i*Reflux	1	12.6914063	12.6914063	3.47	0.0696
Plyslf_m	1	0.1701562	0.1701562	0.05	0.8303
Plyslf_i*Plyslf_m	1	0.3164062	0.3164062	0.09	0.7702
Reflux*Plyslf_m	1	5.7001563	5.7001563	1.56	0.2190
Time	1	142.5039062	142.5039062	38.93	<.0001
Plyslf_i*Time	1	0.3451562	0.3451562	0.09	0.7603
Reflux*Time	1	11.8164062	11.8164062	3.23	0.0796
Plyslf_m*Time	1	2.6001563	2.6001563	0.71	0.4041
Solvent	1	2.7639063	2.7639063	0.76	0.3898
Plyslf_i*Solvent	1	2.7639062	2.7639062	0.76	0.3898
Reflux*Solvent	1	1.7889062	1.7889062	0.49	0.4883
Plyslf_m*Solvent	1	0.0976563	0.0976563	0.03	0.8710
Time*Solvent	1	1.5939062	1.5939062	0.44	0.5129
Temp	1	115.8314062	115.8314062	31.65	<.0001
Plyslf_i*Temp	1	10.8076563	10.8076563	2.95	0.0931
Reflux*Temp	1	0.5076563	0.5076563	0.14	0.7114
Plyslf_m*Temp	1	0.7439063	0.7439063	0.20	0.6544
Time*Temp	1	0.0039062	0.0039062	0.00	0.9741
Solvent*Temp	1	1.1289062	1.1289062	0.31	0.5816

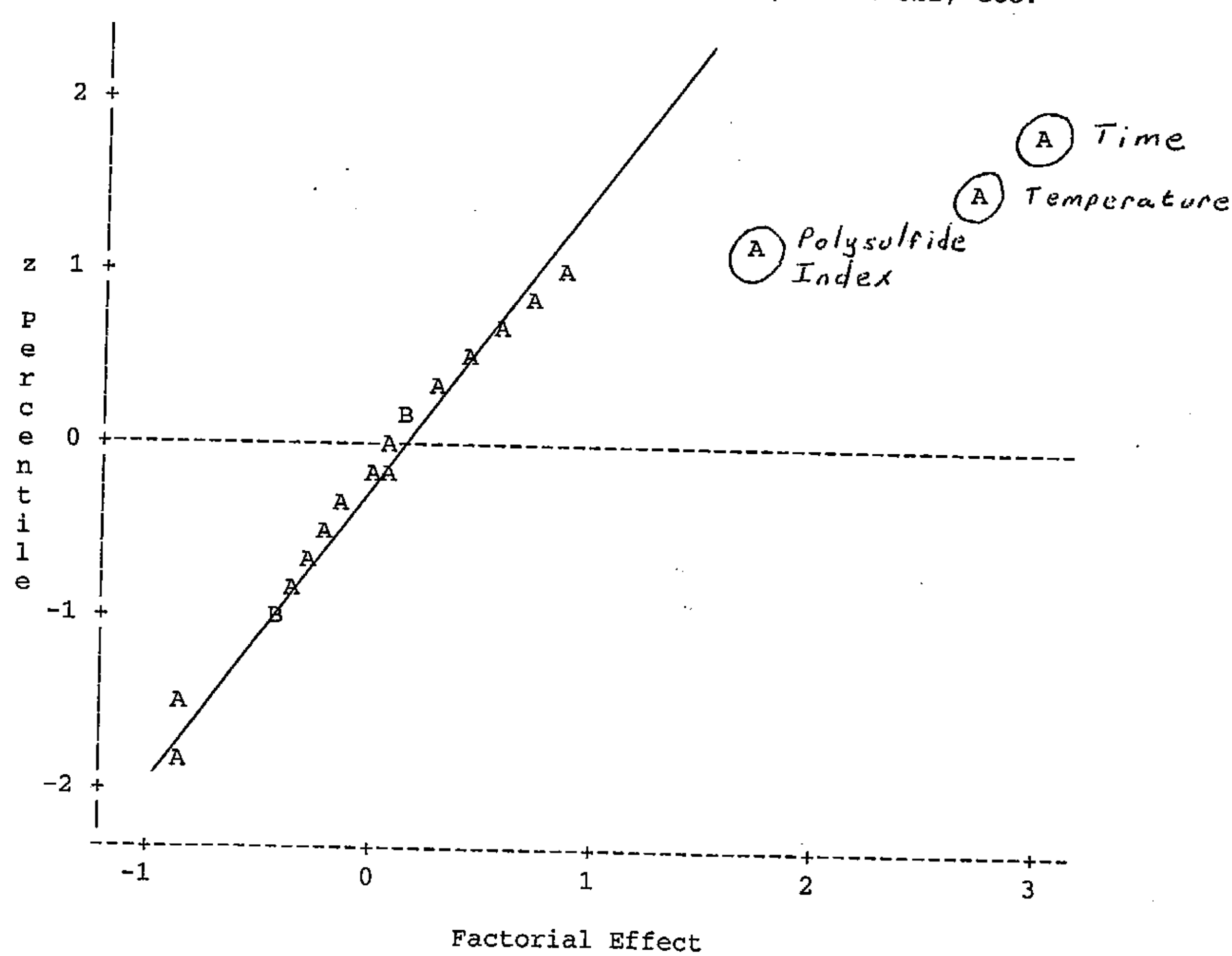
```
*The following code analyzes the full 2^6 factorial experiment when*;
*three-factor and higher-order true interaction effects are assumed*;
*to be zero.*;

Options nodate nonumber ls = 75  ps = 64;

Data Example;

 Do Temp = -1 to 1 by 2; Do Solvent = -1 to 1 by 2; Do Time = -1 to 1 by 2;

 Do Plyslf_m = -1 to 1 by 2; Do Reflux = -1 to 1 by 2;

 Do Plyslf_i = -1 to 1 by 2;

    Input Strength @@; Output;

 End; End; End; End; End; End;

Datalines;

 3.4   9.7   7.4  10.6   6.5   7.9  10.3   9.5  14.3  10.5   7.8  17.2
 9.4  12.1   9.5  15.8   8.3   8.0   7.9  10.7   7.2   7.2   7.9  10.2
10.3   9.9   7.4  10.5   9.6  15.1   8.7  12.1  12.6  10.5  11.3  10.6
 8.1  12.5  11.1  12.9  14.6  12.7  10.8  17.1  13.6  14.6  13.3  14.4
11.0  12.5   8.9  13.1   7.6   8.6  11.8  12.4  13.4  14.6  14.9  11.8
15.6  12.8  13.5  15.8

Proc Glm Data = Example;

Class Plyslf_i Reflux Plyslf_m Time Solvent Temp;

Model Strength = Plyslf_i|Reflux|Plyslf_m|Time|Solvent|Temp@2 / ss3;

Estimate 'Plyslf_i' Plyslf_i -1 1;
Estimate 'Reflux' Reflux -1 1;
Estimate 'Plyslf_m' Plyslf_m  -1 1;
Estimate 'Time' Time -1 1;
Estimate 'Solvent' Solvent -1 1;
Estimate 'Temp' Temp -1 1;

Estimate 'Plyslf_i*Reflux' Plyslf_i*Reflux 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Plyslf_m' Plyslf_i*Plyslf_m 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Time' Plyslf_i*Time 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Solvent' Plyslf_i*Solvent 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Temp' Plyslf_i*Temp 1 -1 -1 1 / divisor = 2;
Estimate 'Reflux*Plyslf_m' Reflux*Plyslf_m 1 -1 -1 1 / divisor = 2;
Estimate 'Reflux*Time' Reflux*Time 1 -1 -1 1 / divisor = 2;
Estimate 'Reflux*Solvent' Reflux*Solvent 1 -1 -1 1 / divisor = 2;
Estimate 'Reflux*Temp' Reflux*Temp 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_m*Time' Plyslf_m*Time 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_m*Solvent' Plyslf_m*Solvent 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_m*Temp' Plyslf_m*Temp 1 -1 -1 1 / divisor = 2;
Estimate 'Time*Solvent' Time*Solvent 1 -1 -1 1 / divisor = 2;
Estimate 'Time*Temp' Time*Temp 1 -1 -1 1 / divisor = 2;
Estimate 'Solvent*Temp' Solvent*Temp 1 -1 -1 1 / divisor = 2;

Title1'The 2^6 Factorial Experiment Analyzed Assuming Three-factor';
Title2'And Higher-order True Interaction Effects Are Zero.';
```

Lee R. Sutton, Jr. Writing Project: Fold-over Designs Data Appendix

```
*The following code constructs a normal probability plot of the*;
*factorial effects from the full 2^6 factorial experiment when*;
*three-factor and higher-order true interaction effects are assumed*;
*to be zero.*;

Data Example2; Input Effects @@; Datalines;

 1.746875   0.703125   0.103125   2.984375  -0.415625   2.690625   0.890625
 0.140625   0.146875  -0.415625  -0.821875   0.596875  -0.859375  -0.334375
-0.178125   0.403125   0.078125  -0.215625  -0.315625   0.015625   0.265625

Proc Rank Data = Example2  Normal = Blom  Out = Fxset;
     Var Effects; Ranks Rankefct;

Proc Plot Data = Fxset  vpercent = 70;
     Label Rankefct = 'z Percentile'  Effects = 'Factorial Effect';
     Plot Rankefct*Effects / vref = 0;

Title1'The 2^6 Factorial Experiment Analyzed Assuming Three-factor';
Title2'And Higher-order True Interaction Effects Are Zero.';
Title3'Normal Probability Plot of the Factorial Effects.';
```

The 2^6 Factorial Experiment Analyzed Assuming Three-factor
And Higher-order True Interaction Effects Are Zero.
Normal Probability Plot of the Factorial Effects.

Plot of Rankefct*Effects. Legend: A = 1 obs, B = 2 obs, etc.



Parameter	Estimate	Standard Error	t Value	Pr > |t|
Plyslf_i	1.74687500	0.47828448	3.65	0.0007
Reflux	0.70312500	0.47828448	1.47	0.1490
Plyslf_m	0.10312500	0.47828448	0.22	0.8303
Time	2.98437500	0.47828448	6.24	<.0001
Solvent	-0.41562500	0.47828448	-0.87	0.3898
Temp	2.69062500	0.47828448	5.63	<.0001
Plyslf_i*Reflux	0.89062500	0.47828448	1.86	0.0696
Plyslf_i*Plyslf_m	0.14062500	0.47828448	0.29	0.7702
Plyslf_i*Time	0.14687500	0.47828448	0.31	0.7603
Plyslf_i*Solvent	-0.41562500	0.47828448	-0.87	0.3898
Plyslf_i*Temp	-0.82187500	0.47828448	-1.72	0.0931
Reflux*Plyslf_m	0.59687500	0.47828448	1.25	0.2190
Reflux*Time	-0.85937500	0.47828448	-1.80	0.0796
Reflux*Solvent	-0.33437500	0.47828448	-0.70	0.4883
Reflux*Temp	-0.17812500	0.47828448	-0.37	0.7114
Plyslf_m*Time	0.40312500	0.47828448	0.84	0.4041
Plyslf_m*Solvent	0.07812500	0.47828448	0.16	0.8710
Plyslf_m*Temp	-0.21562500	0.47828448	-0.45	0.6544
Time*Solvent	-0.31562500	0.47828448	-0.66	0.5129
Time*Temp	0.01562500	0.47828448	0.03	0.9741
Solvent*Temp	0.26562500	0.47828448	0.56	0.5816

```
-1  -1   1  10  11.0
 1  -1   1  39  12.5
-1  -1   1  25   8.9
 1  -1   1  40  13.1
-1  -1   1  30   7.6
 1  -1   1  31   8.6
-1  -1   1  28  11.8
 1  -1   1  49  12.4
-1   1   1  52  13.4
 1   1   1  15  14.6
-1   1   1  34  14.9
 1   1   1  53  11.8
-1   1   1   2  15.6
 1   1   1  12  12.8
-1   1   1  45  13.5
 1   1   1  54  15.8
;

Proc Glm Data = Example;

Model Strength = Plyslf_i Time Temp / ss3;

Output Out = Diag  R = Resid  P = Pred  Rstudent = RSTUDENT;

Title1'The Data Projected into a Replicated 2^3 Factorial Design';
Title2'Involving the Factors Polysulfide Index, Time, and Temperature.';
Title3'All True Interaction Effects Are Assumed to Be Zero.';

Proc Print Data = Diag;

Title1'The Data Projected into a Replicated 2^3 Factorial Design';
Title2'Involving the Factors Polysulfide Index, Time, and Temperature.';
Title3'All True Interaction Effects Are Assumed to Be Zero.';

Proc Plot Data = Diag  vpercent = 85;
     Label RSTUDENT = 'RSTUDENT'  Pred = 'Fitted Value';
     Plot RSTUDENT*Pred / vref = 0;

Title1'The Data Projected into a Replicated 2^3 Factorial Design';
Title2'Involving the Factors Polysulfide Index, Time, and Temperature.';
Title3'All True Interaction Effects Are Assumed to Be Zero.';
Title4'Plot of RSTUDENT against the Fitted Values.';

Proc Plot Data = Diag  vpercent = 85;
     Label Resid = 'Residual'  Pred = 'Fitted Value';
     Plot Resid*Pred / vref = 0;

Title1'The Data Projected into a Replicated 2^3 Factorial Design';
Title2'Involving the Factors Polysulfide Index, Time, and Temperature.';
Title3'All True Interaction Effects Are Assumed to Be Zero.';
Title4'Plot of the Residuals against the Fitted Values.';

Proc Plot Data = Diag  vpercent = 85;
     Label Resid = 'Residual'  Plyslf_i = 'Polysulfide Index';
     Plot Resid*Plyslf_i / vref = 0;

Title1'The Data Projected into a Replicated 2^3 Factorial Design';
Title2'Involving the Factors Polysulfide Index, Time, and Temperature.';
Title3'All True Interaction Effects Are Assumed to Be Zero.';
Title4'Plot of the Residuals against the Levels of Polysulfide Index.';
```

Lee R. Sutton, Jr. Writing Project: Fold-over Designs Data Appendix

```
*The following code projects the data from the 2^6 factorial*;
*experiment into a replicated 2^3 factorial design involving the*;
*factors Polysulfide Index, Time, and Temperature and makes residual*;
*plots to check model assumptions.  All true interaction effects are*;
*assumed to be zero and the design is replicated eight times.*;

Data Example;

Input Plyslf_i Time Temp Order Strength;

Datalines;

-1  -1  -1  26   3.4
 1  -1  -1   3   9.7
-1  -1  -1  11   7.4
 1  -1  -1   5  10.6
-1  -1  -1  42   6.5
 1  -1  -1  18   7.9
-1  -1  -1  41  10.3
 1  -1  -1  14   9.5
-1   1  -1  17  14.3
 1   1  -1  27  10.5
-1   1  -1  19   7.8
 1   1  -1  56  17.2
-1   1  -1  23   9.4
 1   1  -1   8  12.1
-1   1  -1  32   9.5
 1   1  -1   7  15.8
-1  -1  -1  46   8.3
 1  -1  -1  13   8.0
-1  -1  -1  58   7.9
 1  -1  -1  38  10.7
-1  -1  -1  43   7.2
 1  -1  -1  55   7.2
-1  -1  -1   6   7.9
 1  -1  -1  64  10.2
-1   1  -1  22  10.3
 1   1  -1   4   9.9
-1   1  -1  16   7.4
 1   1  -1  47  10.5
-1   1  -1  63   9.6
 1   1  -1  51  15.1
-1   1  -1  20   8.7
 1   1  -1  29  12.1
-1  -1   1  62  12.6
 1  -1   1   1  10.5
-1  -1   1  37  11.3
 1  -1   1  61  10.6
-1  -1   1  44   8.1
 1  -1   1  24  12.5
-1  -1   1  59  11.1
 1  -1   1  60  12.9
-1   1   1  35  14.6
 1   1   1  50  12.7
-1   1   1  48  10.8
 1   1   1  36  17.1
-1   1   1  21  13.6
 1   1   1   9  14.6
-1   1   1  33  13.3
 1   1   1  57  14.4
```

Lee R. Sutton, Jr. Writing Project: Fold-over Designs Data Appendix

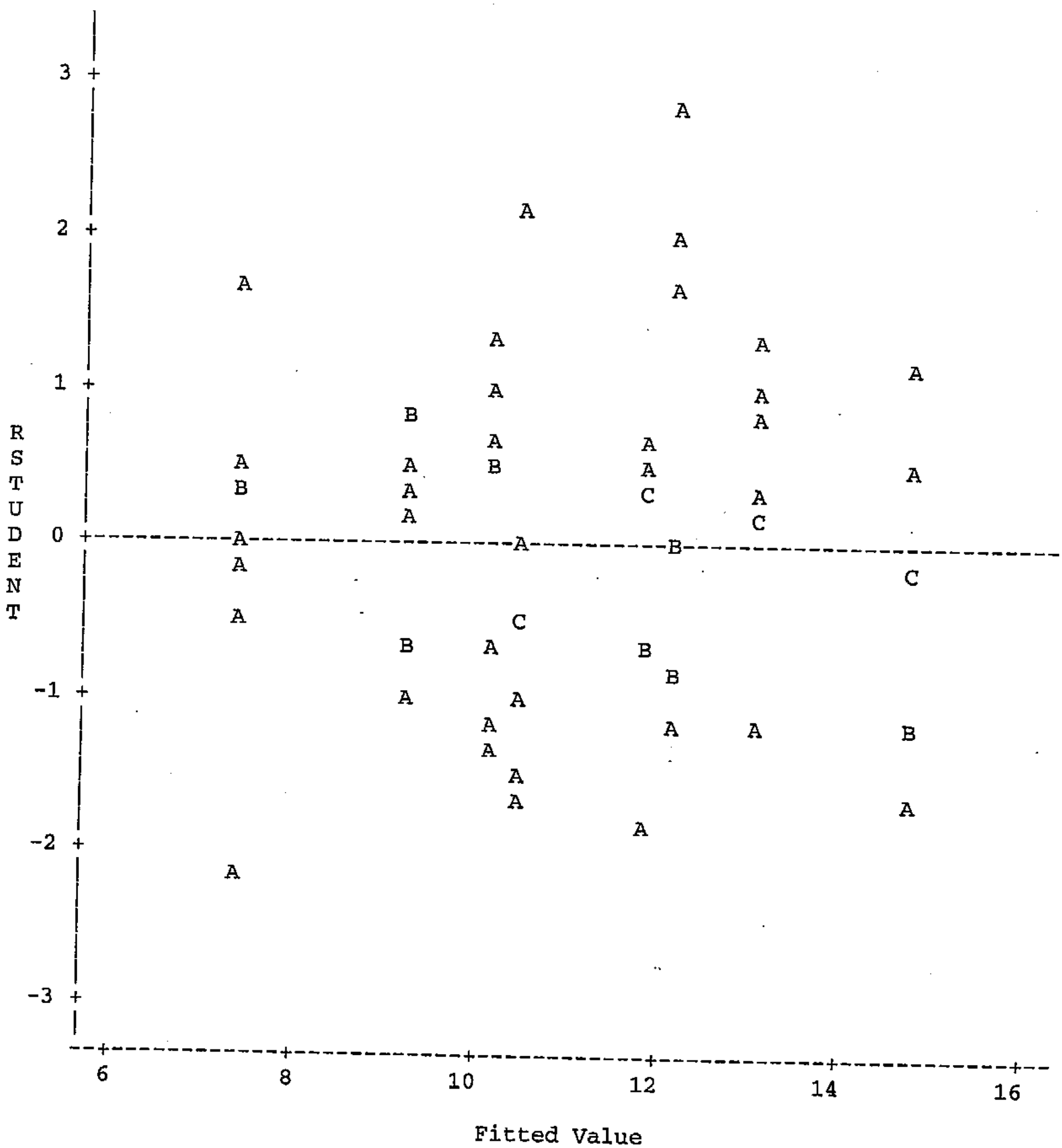
The Data Projected into a Replicated 2^3 Factorial Design
Involving the Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.

Obs	Plyslf_i	Time	Temp	Order	Strength	Resid	Pred	RSTUDENT
1	-1	-1	-1	26	3.4	-4.01250	7.4125	-2.24913
2	1	-1	-1	3	9.7	0.54063	9.1594	0.29103
3	-1	-1	-1	11	7.4	-0.01250	7.4125	-0.00672
4	1	-1	-1	5	10.6	1.44063	9.1594	0.77895
5	-1	-1	-1	42	6.5	-0.91250	7.4125	-0.49188
6	1	-1	-1	18	7.9	-1.25937	9.1594	-0.68012
7	-1	-1	-1	41	10.3	2.88750	7.4125	1.58608
8	1	-1	-1	14	9.5	0.34063	9.1594	0.18329
9	-1	1	-1	17	14.3	3.90313	10.3969	2.18279
10	1	1	-1	27	10.5	-1.64375	12.1438	-0.89016
11	-1	1	-1	19	7.8	-2.59688	10.3969	-1.42066
12	1	1	-1	56	17.2	5.05625	12.1438	2.90843
13	-1	1	-1	23	9.4	-0.99687	10.3969	-0.53757
14	1	1	-1	8	12.1	-0.04375	12.1438	-0.02354
15	-1	1	-1	32	9.5	-0.89687	10.3969	-0.48342
16	1	1	-1	7	15.8	3.65625	12.1438	2.03469
17	-1	-1	-1	46	8.3	0.88750	7.4125	0.47835
18	1	-1	-1	13	8.0	-1.15937	9.1594	-0.62574
19	-1	-1	-1	58	7.9	0.48750	7.4125	0.26240
20	1	-1	-1	38	10.7	1.54063	9.1594	0.83364
21	-1	-1	-1	43	7.2	-0.21250	7.4125	-0.11433
22	1	-1	-1	55	7.2	-1.95937	9.1594	-1.06410
23	-1	-1	-1	6	7.9	0.48750	7.4125	0.26240
24	1	-1	-1	64	10.2	1.04063	9.1594	0.56129
25	-1	1	-1	22	10.3	-0.09687	10.3969	-0.05211
26	1	1	-1	4	9.9	-2.24375	12.1438	-1.22219
27	-1	1	-1	16	7.4	-2.99687	10.3969	-1.64887
28	1	1	-1	47	10.5	-1.64375	12.1438	-0.89016
29	-1	1	-1	63	9.6	-0.79688	10.3969	-0.42934
30	1	1	-1	51	15.1	2.95625	12.1438	1.62551
31	-1	1	-1	20	8.7	-1.69688	10.3969	-0.91934
32	1	1	-1	29	12.1	-0.04375	12.1438	-0.02354
33	-1	-1	1	62	12.6	2.49688	10.1031	1.36420
34	1	-1	1	1	10.5	-1.35000	11.8500	-0.72949
35	-1	-1	1	37	11.3	1.19688	10.1031	0.64612
36	1	-1	1	61	10.6	-1.25000	11.8500	-0.67502
37	-1	-1	1	44	8.1	-2.00312	10.1031	-1.08833
38	1	-1	1	24	12.5	0.65000	11.8500	0.35003
39	-1	-1	1	59	11.1	0.99688	10.1031	0.53757
40	1	-1	1	60	12.9	1.05000	11.8500	0.56637
41	-1	1	1	35	14.6	1.51250	13.0875	0.81824
42	1	1	1	50	12.7	-2.13437	14.8344	-1.16122
43	-1	1	1	48	10.8	-2.28750	13.0875	-1.24665
44	1	1	1	36	17.1	2.26563	14.8344	1.23442
45	-1	1	1	21	13.6	0.51250	13.0875	0.27587
46	1	1	1	9	14.6	-0.23437	14.8344	-0.12610
47	-1	1	1	33	13.3	0.21250	13.0875	0.11433
48	1	1	1	57	14.4	-0.43437	14.8344	-0.23378
49	-1	-1	1	10	11.0	0.89688	10.1031	0.48342
50	1	-1	1	39	12.5	0.65000	11.8500	0.35003
51	-1	-1	1	25	8.9	-1.20312	10.1031	-0.64952
52	1	-1	1	40	13.1	1.25000	11.8500	0.67502
53	-1	-1	1	30	7.6	-2.50312	10.1031	-1.36772
54	1	-1	1	31	8.6	-3.25000	11.8500	-1.79544
55	-1	-1	1	28	11.8	1.69688	10.1031	0.91934

```
Proc Plot Data = Diag  vpercent = 85;
     Label Resid = 'Residual'  Time = 'Time';
     Plot Resid*Time / vref = 0;

Title1'The Data Projected into a Replicated 2^3 Factorial Design';
Title2'Involving the Factors Polysulfide Index, Time, and Temperature.';
Title3'All True Interaction Effects Are Assumed to Be Zero.';
Title4'Plot of the Residuals against the Levels of Time.';

Proc Plot Data = Diag  vpercent = 85;
     Label Resid = 'Residual'  Temp = 'Temperature';
     Plot Resid*Temp / vref = 0;

Title1'The Data Projected into a Replicated 2^3 Factorial Design';
Title2'Involving the Factors Polysulfide Index, Time, and Temperature.';
Title3'All True Interaction Effects Are Assumed to Be Zero.';
Title4'Plot of the Residuals against the Levels of Temperature.';

Proc Plot Data = Diag  vpercent = 85;
     Label Resid = 'Residual'  Order = 'Time Order of Observation';
     Plot Resid*Order / vref = 0;

Title1'The Data Projected into a Replicated 2^3 Factorial Design';
Title2'Involving the Factors Polysulfide Index, Time, and Temperature.';
Title3'All True Interaction Effects Are Assumed to Be Zero.';
Title4'Plot of the Residuals against the Time Order of the Observations.';

Proc Rank Data = Diag  Normal = Blom  Out = Normset;
     Var RSTUDENT; Ranks Rankres;

Proc Plot Data = Normset  vpercent = 85;
     Label Rankres = 'z Percentile'  RSTUDENT = 'RSTUDENT';
     Plot Rankres*RSTUDENT / vref = 0;

Title1'The Data Projected into a Replicated 2^3 Factorial Design';
Title2'Involving the Factors Polysulfide Index, Time, and Temperature.';
Title3'All True Interaction Effects Are Assumed to Be Zero.';
Title4'Normal Probability Plot of RSTUDENT.';

Proc Rank Data = Diag  Normal = Blom  Out = Normset;
     Var Resid; Ranks Rankres;

Proc Plot Data = Normset  vpercent = 85;
     Label Rankres = 'z Percentile'  Resid = 'Residual';
     Plot Rankres*Resid / vref = 0;

Title1'The Data Projected into a Replicated 2^3 Factorial Design';
Title2'Involving the Factors Polysulfide Index, Time, and Temperature.';
Title3'All True Interaction Effects Are Assumed to Be Zero.';
Title4'Normal Probability Plot of the Residuals.';
```

The Data Projected into a Replicated 2^3 Factorial Design
Involving the Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.
Plot of RSTUDENT against the Fitted Values.

Plot of RSTUDENT*Pred. Legend: A = 1 obs, B = 2 obs, etc.



The Data Projected into a Replicated 2^3 Factorial Design
Involving the Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.

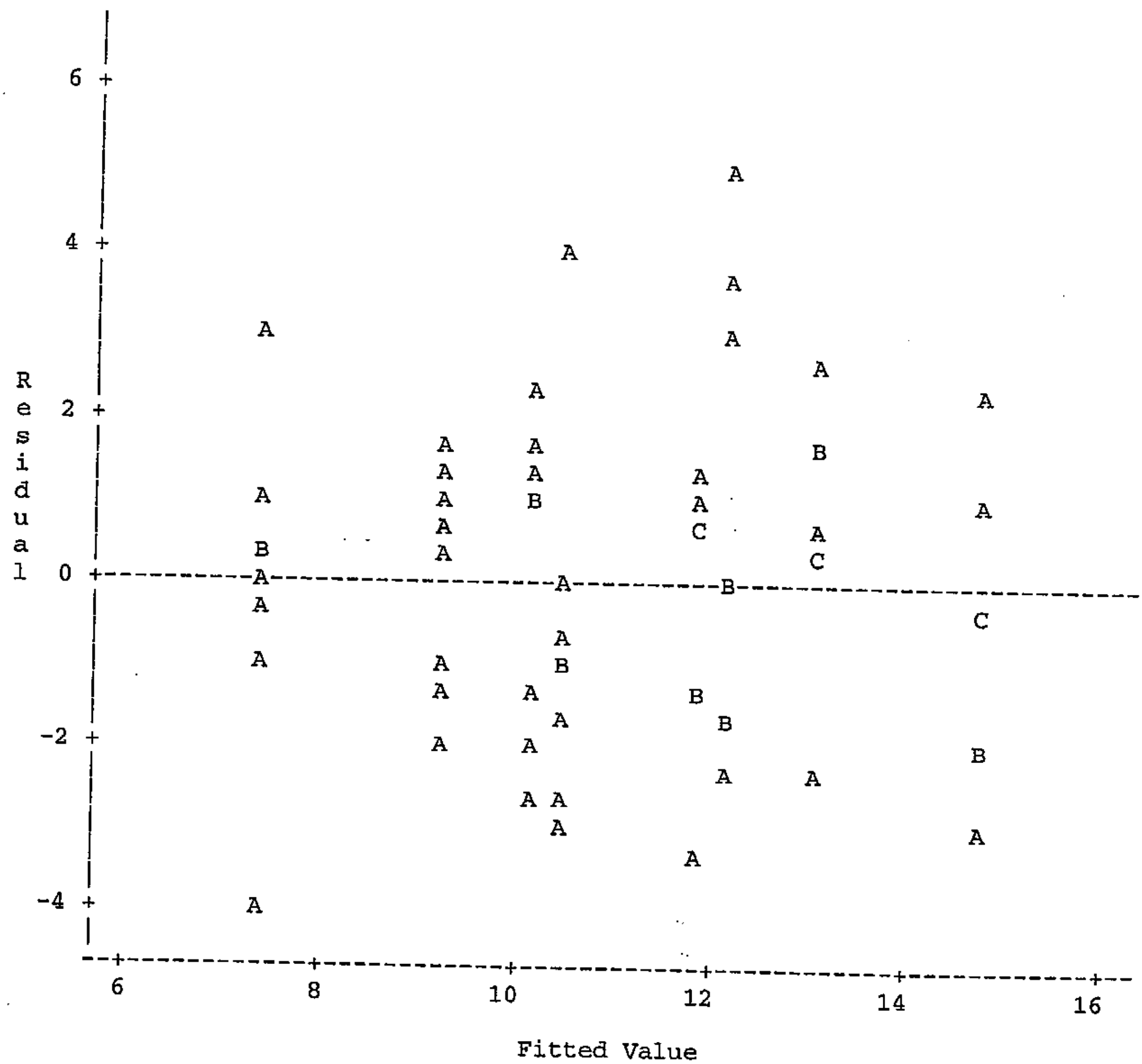
Obs	Plyslf_i	Time	Temp	Order	Strength	Resid	Pred	RSTUDENT
56	1	-1	1	49	12.4	0.55000	11.8500	0.29609
57	-1	1	1	52	13.4	0.31250	13.0875	0.16815
58	1	1	1	15	14.6	-0.23437	14.8344	-0.12610
59	-1	1	1	34	14.9	1.81250	13.0875	0.98297
60	1	1	1	53	11.8	-3.03437	14.8344	-1.67048
61	-1	1	1	2	15.6	2.51250	13.0875	1.37300
62	1	1	1	12	12.8	-2.03437	14.8344	-1.10566
63	-1	1	1	45	13.5	0.41250	13.0875	0.22199
64	1	1	1	54	15.8	0.96563	14.8344	0.52064

The Data Projected into a Replicated 2^3 Factorial Design
Involving the Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.
Plot of the Residuals against the Levels of Polysulfide Index.

```
  Plot of Resid*Plyslf_i.  Legend: A = 1 obs, B = 2 obs, etc.

        |
      6 +
        |
        |
        |                                                A
        |
        |
      4 +  A
        |                                                A
        |
        |  A                                             A
        |  A
 R      |  A                                             A
 e    2 +
 s      |  C                                             A
 i      |  A                                             B
 d      |  C                                             C
 u      |  A                                             D
 a      |  E                                             A
 l    0 +--B---------------------------------------------B--
        |  A                                             C
        |  A
        |  C                                             A
        |  A                                             C
        |  A                                             B
     -2 +  A                                             C
        |  A                                             A
        |  B
        |  A                                             A
        |                                                A
        |
     -4 +  A
        |
        ---+---------------------------------------------+--
          -1                                             1

                           Polysulfide Index
```

The Data Projected into a Replicated 2^3 Factorial Design
Involving the Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.
Plot of the Residuals against the Fitted Values.

Plot of Resid*Pred. Legend: A = 1 obs, B = 2 obs, etc.



The Data Projected into a Replicated 2^3 Factorial Design
Involving the Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.
Plot of the Residuals against the Levels of Temperature.

```
         Plot of Resid*Temp.  Legend: A = 1 obs, B = 2 obs, etc.

       |
       |
     6 +
       |
       |
       |  A
       |
       |
     4 +  A
       |  A
       |
       |  B
       |                                                     A
 R     |                                                     B
 e   2 +
 s     |  A                                                  C
 i     |  A                                                  B
 d     |  B                                                  D
 u     |  A                                                  D
 a     |  C                                                  C
 l   0 +--D------------------------------------------------------
       |  A                                                  C
       |  A
       |  D
       |  A                                                  C
       |  C
    -2 +  A                                                  C
       |  A                                                  A
       |  A                                                  A
       |  A                                                  A
       |                                                     A
       |
    -4 +  A
       |
       ---+--------------------------------------------------+--
         -1                                                  1

                                  Temperature
```

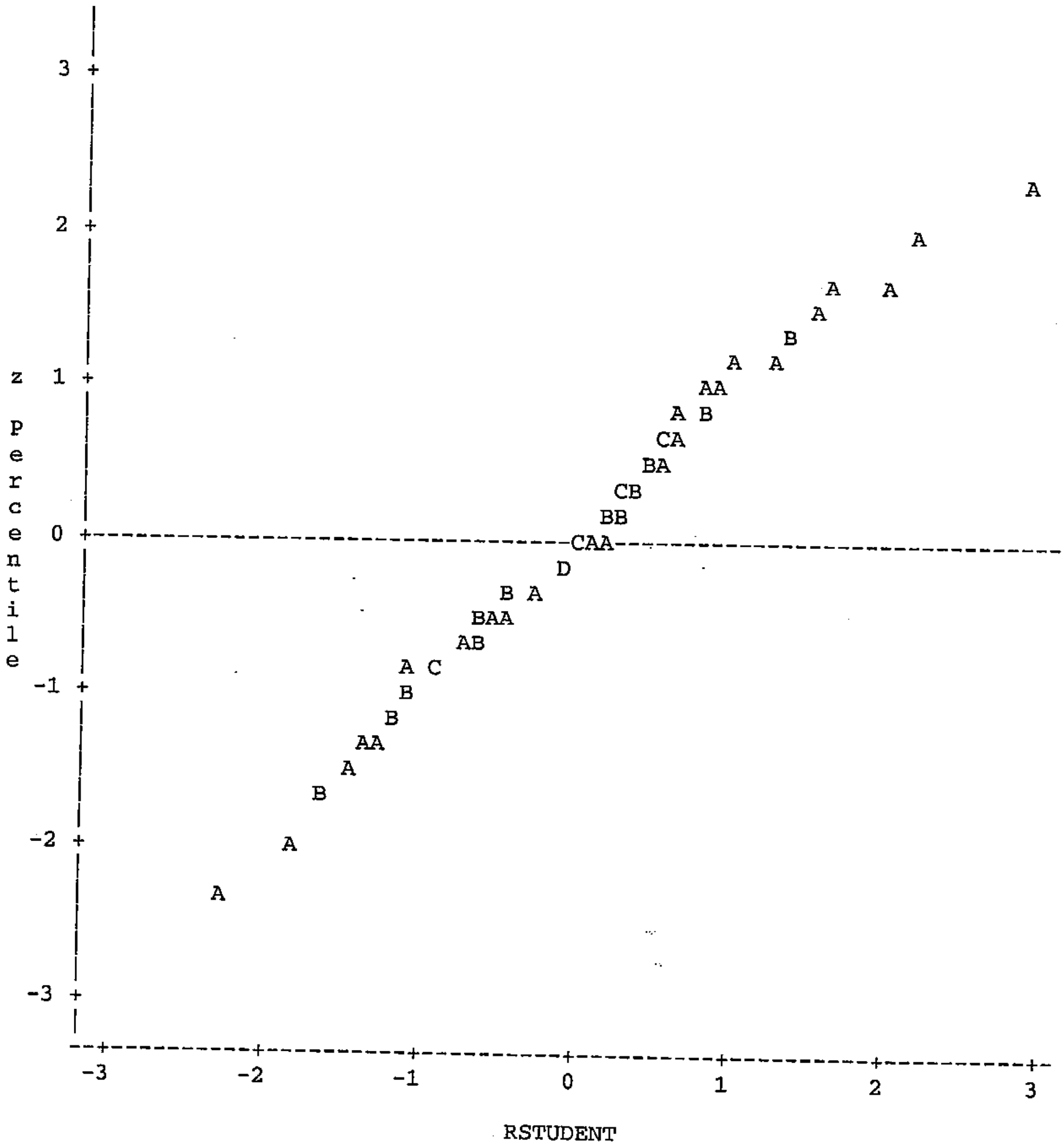
The Data Projected into a Replicated 2^3 Factorial Design
Involving the Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.
Plot of the Residuals against the Levels of Time.

Plot of Resid*Time. Legend: A = 1 obs, B = 2 obs, etc.

```
        |
      6 +
        |
        |
        |                                                     A
        |
        |
      4 +                                                     A
        |                                                     A
        |
        |  A                                                  A
        |                                                     A
  R     |  A                                                  A
  e   2 +
  s     |  B                                                  B
  i     |  C
  d     |  E                                                  A
  u     |  D                                                  A
  a     |  C                                                  C
  l   0 +--A--------------------------------------------------C--
        |  A                                                  C
        |                                                     A
        |  B                                                  B
        |  D
        |                                                     C
     -2 +  B                                                  B
        |                                                     B
        |  A                                                  A
        |                                                     B
        |  A
        |
     -4 +  A
        |
        ---+--------------------------------------------------+--
          -1                                                  1

                                 Time
```

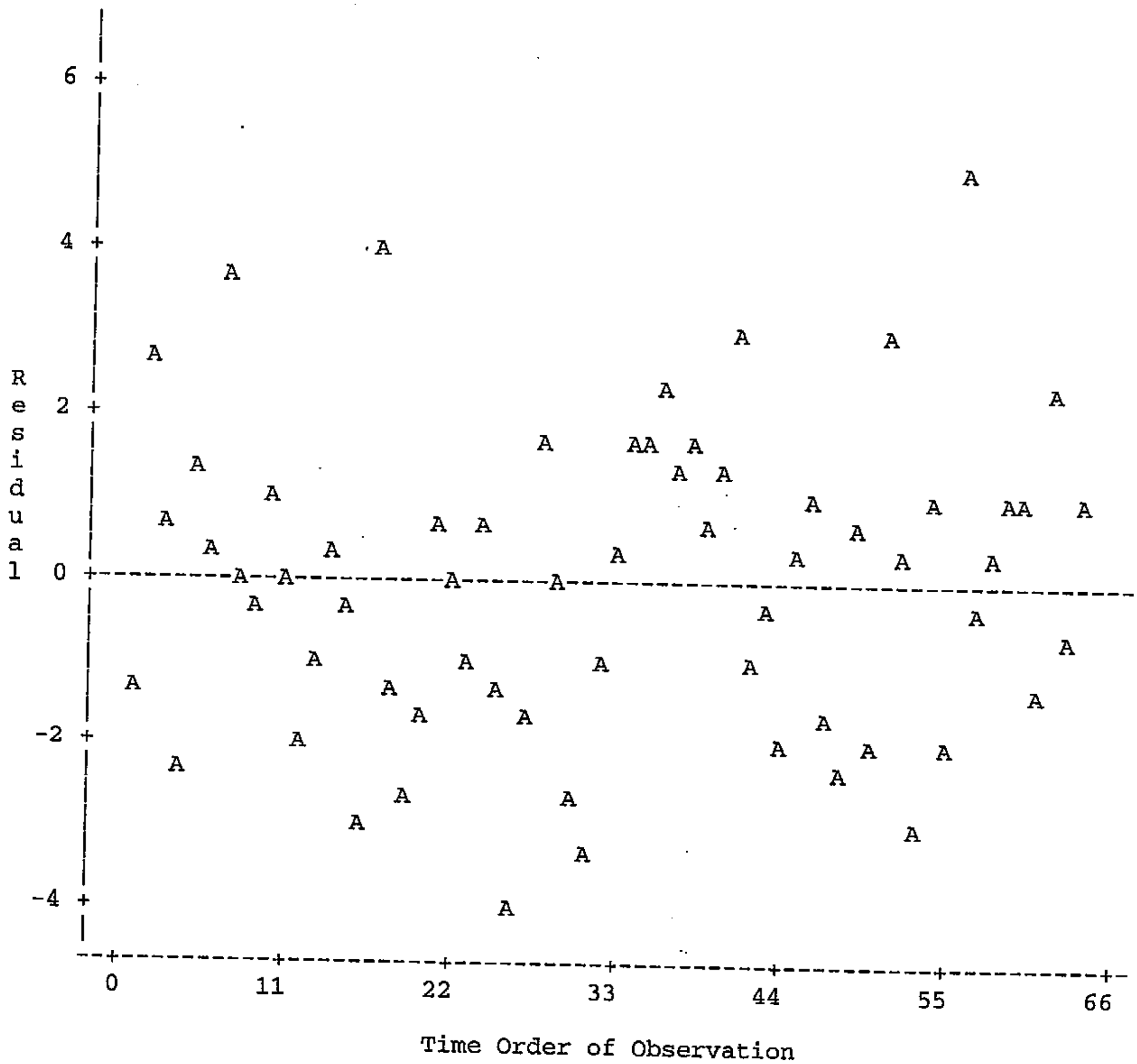
The Data Projected into a Replicated 2^3 Factorial Design
Involving the Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.
Normal Probability Plot of RSTUDENT.

Plot of Rankres*RSTUDENT. Legend: A = 1 obs, B = 2 obs, etc.



The Data Projected into a Replicated 2^3 Factorial Design
Involving the Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.
Plot of the Residuals against the Time Order of the Observations.

Plot of Resid*Order. Legend: A = 1 obs, B = 2 obs, etc.



The Data Projected into a Replicated 2^3 Factorial Design
Involving the Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.

The GLM Procedure

Dependent Variable: Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	307.1604687	102.3868229	28.25	<.0001
Error	60	217.4743750	3.6245729		
Corrected Total	63	524.6348438			

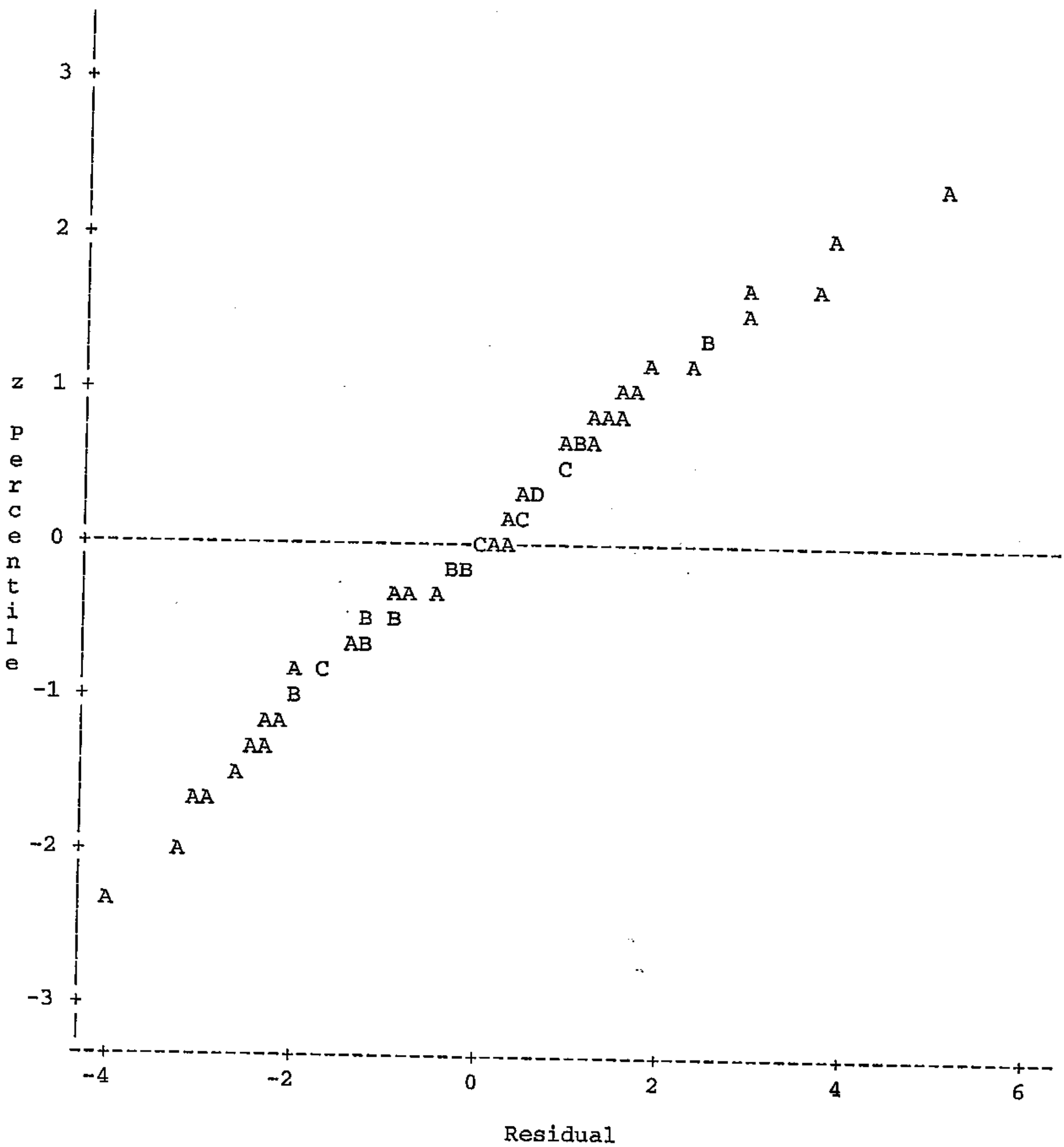
R-Square	Coeff Var	Root MSE	Strength Mean
0.585475	17.11549	1.903831	11.12344

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Plyslf_i	1	48.8251563	48.8251563	13.47	0.0005
Time	1	142.5039063	142.5039063	39.32	<.0001
Temp	1	115.8314062	115.8314062	31.96	<.0001

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Intercept	11.12343750	0.23797889	46.74	<.0001
Plyslf_i	0.87343750	0.23797889	3.67	0.0005
Time	1.49218750	0.23797889	6.27	<.0001
Temp	1.34531250	0.23797889	5.65	<.0001

The Data Projected into a Replicated 2^3 Factorial Design
Involving the Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.
Normal Probability Plot of the Residuals.

Plot of Rankres*Resid. Legend: A = 1 obs, B = 2 obs, etc.



All 1/8 Fractions of a 2^6 Factorial Design
Using +-ABD, +-ACE, and +-BCF as Design Generators.

------------------------------- FRACTION=1 --------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
1	1	-1	1	-1	-1	1	1	1
-1	-1	1	1	-1	-1	1	1	1
-1	1	-1	-1	1	-1	1	1	1
1	-1	1	-1	1	-1	1	1	1
1	-1	-1	-1	-1	1	1	1	1
-1	1	1	-1	-1	1	1	1	1
-1	-1	-1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1

------------------------------- FRACTION=2 --------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
1	1	-1	-1	-1	-1	-1	1	1
-1	-1	1	-1	-1	-1	-1	1	1
-1	1	-1	1	1	-1	-1	1	1
1	-1	1	1	1	-1	-1	1	1
1	-1	-1	1	-1	1	-1	1	1
-1	1	1	1	-1	1	-1	1	1
-1	-1	-1	-1	1	1	-1	1	1
1	1	1	-1	1	1	-1	1	1

------------------------------- FRACTION=3 --------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
-1	1	-1	-1	-1	-1	1	-1	1
1	-1	1	-1	-1	-1	1	-1	1
1	1	-1	1	1	-1	1	-1	1
-1	-1	1	1	1	-1	1	-1	1
-1	-1	-1	1	-1	1	1	-1	1
1	1	1	1	-1	1	1	-1	1
1	-1	-1	-1	1	1	1	-1	1
-1	1	1	-1	1	1	1	-1	1

------------------------------- FRACTION=4 --------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
1	-1	-1	-1	-1	-1	1	1	-1
-1	1	1	-1	-1	-1	1	1	-1
-1	-1	-1	1	1	-1	1	1	-1
1	1	1	1	1	-1	1	1	-1
1	1	-1	1	-1	1	1	1	-1
-1	-1	1	1	-1	1	1	1	-1
-1	1	-1	-1	1	1	1	1	-1
1	-1	1	-1	1	1	1	1	-1

```
*The following code generates all 1/8 fractions of a 2^6 factorial*;
*design using +-ABD, +-ACE, and +-BCF as design generators.*;

Data Example;

Do  F = -1 to 1 by 2;
Do  E = -1 to 1 by 2;
Do  D = -1 to 1 by 2;
Do  C = -1 to 1 by 2;
Do  B = -1 to 1 by 2;
Do  A = -1 to 1 by 2;

DR1 = A*B*D; DR2 = A*C*E; DR3 = B*C*F;

If  DR1 =  1  And  DR2 =  1  And  DR3 =  1  Then  FRACTION = 1;
If  DR1 = -1  And  DR2 =  1  And  DR3 =  1  Then  FRACTION = 2;
If  DR1 =  1  And  DR2 = -1  And  DR3 =  1  Then  FRACTION = 3;
If  DR1 =  1  And  DR2 =  1  And  DR3 = -1  Then  FRACTION = 4;
If  DR1 = -1  And  DR2 = -1  And  DR3 =  1  Then  FRACTION = 5;
If  DR1 = -1  And  DR2 =  1  And  DR3 = -1  Then  FRACTION = 6;
If  DR1 =  1  And  DR2 = -1  And  DR3 = -1  Then  FRACTION = 7;
If  DR1 = -1  And  DR2 = -1  And  DR3 = -1  Then  FRACTION = 8; Output;

End; End; End; End; End; End;

Proc Sort Data = Example; By FRACTION;
Proc Print Data = Example; By FRACTION;
     ID A B C D E F; Var DR1 DR2 DR3;

Title'All 1/8 Fractions of a 2^6 Factorial Design';
Title2'Using +-ABD, +-ACE, and +-BCF as Design Generators.';
```

```
*The following code analyzes the first half of Fold-over Design 1.*;
*The design generators for this half are ABD, ACE, and BCF.*;

Data Example;

 Do Plyslf_m = -1 to 1 by 2; Do Reflux = -1 to 1 by 2;

 Do Plyslf_i = -1 to 1 by 2;

 Time = Plyslf_i*Reflux; Solvent = Plyslf_i*Plyslf_m;

 Temp = Reflux*Plyslf_m;

    Input Strength @@; Output;

 End; End; End;

Datalines;

13.4  10.5  7.9  17.2  9.4  7.2  11.1  15.8

Proc Glm Data = Example;

Class Plyslf_i Reflux Plyslf_m Time Solvent Temp;

Model Strength = Plyslf_i Reflux Plyslf_m Time Solvent Temp

      Plyslf_i*Temp / ss3;

Estimate 'Plyslf_i' Plyslf_i -1 1;
Estimate 'Reflux' Reflux -1 1;
Estimate 'Plyslf_m' Plyslf_m  -1 1;
Estimate 'Time' Time -1 1;
Estimate 'Solvent' Solvent -1 1;
Estimate 'Temp' Temp -1 1;

Estimate 'Plyslf_i*Temp' Plyslf_i*Temp 1 -1 -1 1 / divisor = 2;

Title'The First Half of Fold-over Design 1.';
```

------------------------------- FRACTION=5 --------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
-1	1	-1	1	-1	-1	-1	-1	1
1	-1	1	1	-1	-1	-1	-1	1
1	1	-1	-1	1	-1	-1	-1	1
-1	-1	1	-1	1	-1	-1	-1	1
-1	-1	-1	-1	-1	1	-1	-1	1
1	1	1	-1	-1	1	-1	-1	1
1	-1	-1	1	1	1	-1	-1	1
-1	1	1	1	1	1	-1	-1	1

------------------------------- FRACTION=6 --------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
1	-1	-1	1	-1	-1	-1	1	-1
-1	1	1	1	-1	-1	-1	1	-1
-1	-1	-1	-1	1	-1	-1	1	-1
1	1	1	-1	1	-1	-1	1	-1
1	1	-1	-1	-1	1	-1	1	-1
-1	-1	1	-1	-1	1	-1	1	-1
-1	1	-1	1	1	1	-1	1	-1
1	-1	1	1	1	1	-1	1	-1

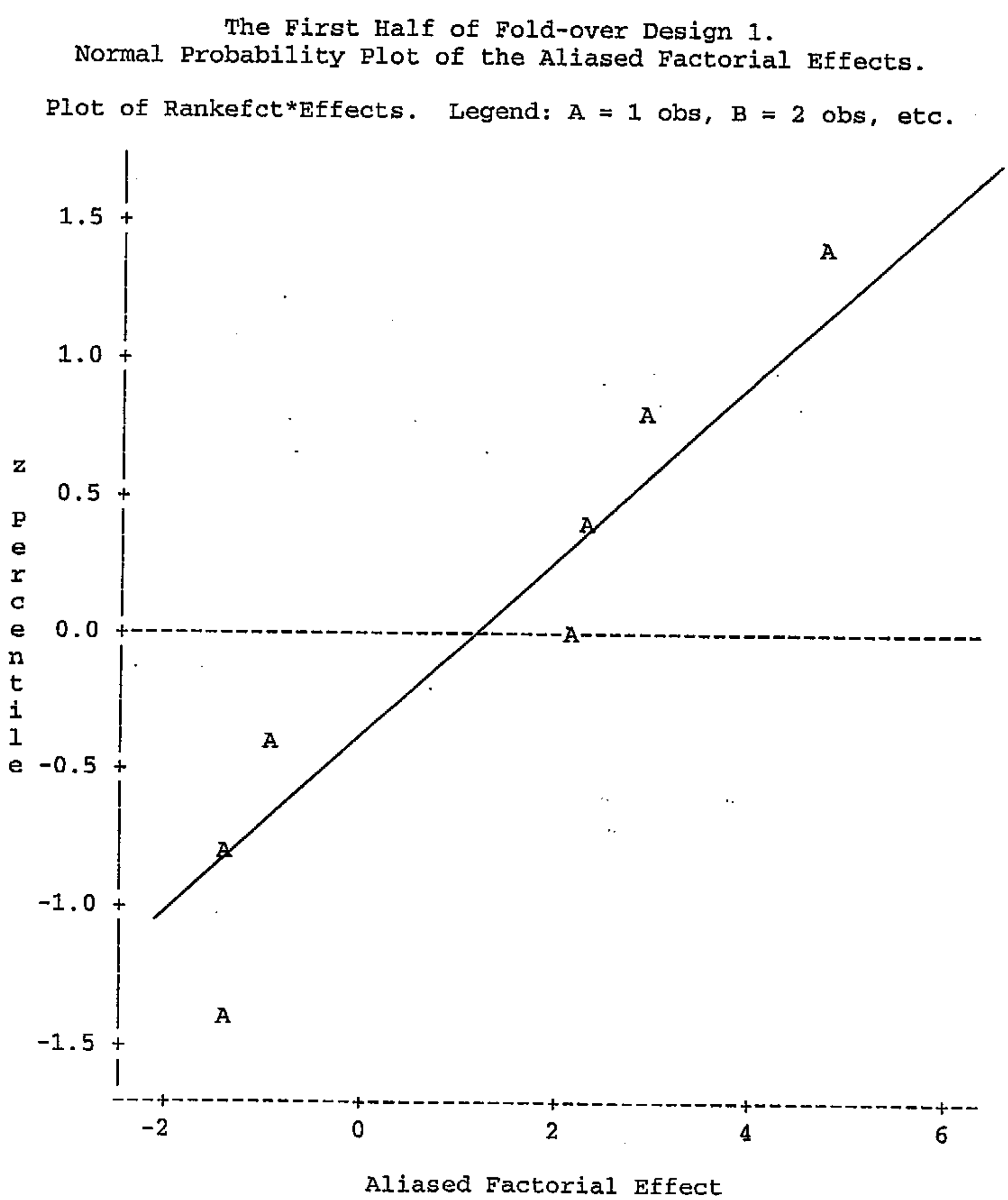
------------------------------- FRACTION=7 --------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
-1	-1	-1	1	-1	-1	1	-1	-1
1	1	1	1	-1	-1	1	-1	-1
1	-1	-1	-1	1	-1	1	-1	-1
-1	1	1	-1	1	-1	1	-1	-1
-1	1	-1	-1	-1	1	1	-1	-1
1	-1	1	-1	-1	1	1	-1	-1
1	1	-1	1	1	1	1	-1	-1
-1	-1	1	1	1	1	1	-1	-1

------------------------------- FRACTION=8 --------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
-1	-1	-1	-1	-1	-1	-1	-1	-1
1	1	1	-1	-1	-1	-1	-1	-1
1	-1	-1	1	1	-1	-1	-1	-1
-1	1	1	1	1	-1	-1	-1	-1
-1	1	-1	1	-1	1	-1	-1	-1
1	-1	1	1	-1	1	-1	-1	-1
1	1	-1	-1	1	1	-1	-1	-1
-1	-1	1	-1	1	1	-1	-1	-1

```
*The following code constructs a normal probability plot of the*;
*aliased factorial effects from the first half of Fold-over*;
*Design 1.*;

Data Example3; Input Effects @@; Datalines;

2.225  2.875  -1.375  4.775  -0.975  2.275  -1.325

Proc Rank Data = Example3  Normal = Blom  Out = Fxset;
     Var Effects; Ranks Rankefct;

Proc Plot Data = Fxset  vpercent = 70;
     Label Rankefct = 'z Percentile'  Effects = 'Aliased Factorial Effect';
     Plot Rankefct*Effects / vref = 0;

Title1'The First Half of Fold-over Design 1.';
Title2'Normal Probability Plot of the Aliased Factorial Effects.';
```



The First Half of Fold-over Design 1.

The GLM Procedure

Dependent Variable: Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	91.57875000	13.08267857	.	.
Error	0	0.00000000	.		
Corrected Total	7	91.57875000			

R-Square	Coeff Var	Root MSE	Strength Mean
1.000000	.	.	11.56250

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Plyslf_i	1	9.90125000	9.90125000	.	.
Reflux	1	16.53125000	16.53125000	.	.
Plyslf_m	1	3.78125000	3.78125000	.	.
Time	1	45.60125000	45.60125000	.	.
Solvent	1	1.90125000	1.90125000	.	.
Temp	1	10.35125000	10.35125000	.	.
Plyslf_i*Temp	1	3.51125000	3.51125000	.	.

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Plyslf_i	2.22500000	.	.	.
Reflux	2.87500000	.	.	.
Plyslf_m	-1.37500000	.	.	.
Time	4.77500000	.	.	.
Solvent	-0.97500000	.	.	.
Temp	2.27500000	.	.	.
Plyslf_i*Temp	-1.32500000	.	.	.

The Second Half of Fold-over Design 1.

The GLM Procedure

Dependent Variable: Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	81.96000000	11.70857143	.	.
Error	0	0.00000000	.		
Corrected Total	7	81.96000000			

R-Square	Coeff Var	Root MSE	Strength Mean
1.000000	.	.	9.700000

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Plyslf_i	1	34.44500000	34.44500000	.	.
Reflux	1	5.44500000	5.44500000	.	.
Plyslf_m	1	1.28000000	1.28000000	.	.
Time	1	13.52000000	13.52000000	.	.
Solvent	1	0.12500000	0.12500000	.	.
Temp	1	26.64500000	26.64500000	.	.
Plyslf_i*Temp	1	0.50000000	0.50000000	.	.

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Plyslf_i	4.15000000	.	.	.
Reflux	1.65000000	.	.	.
Plyslf_m	0.80000000	.	.	.
Time	2.60000000	.	.	.
Solvent	0.25000000	.	.	.
Temp	3.65000000	.	.	.
Plyslf_i*Temp	0.50000000	.	.	.

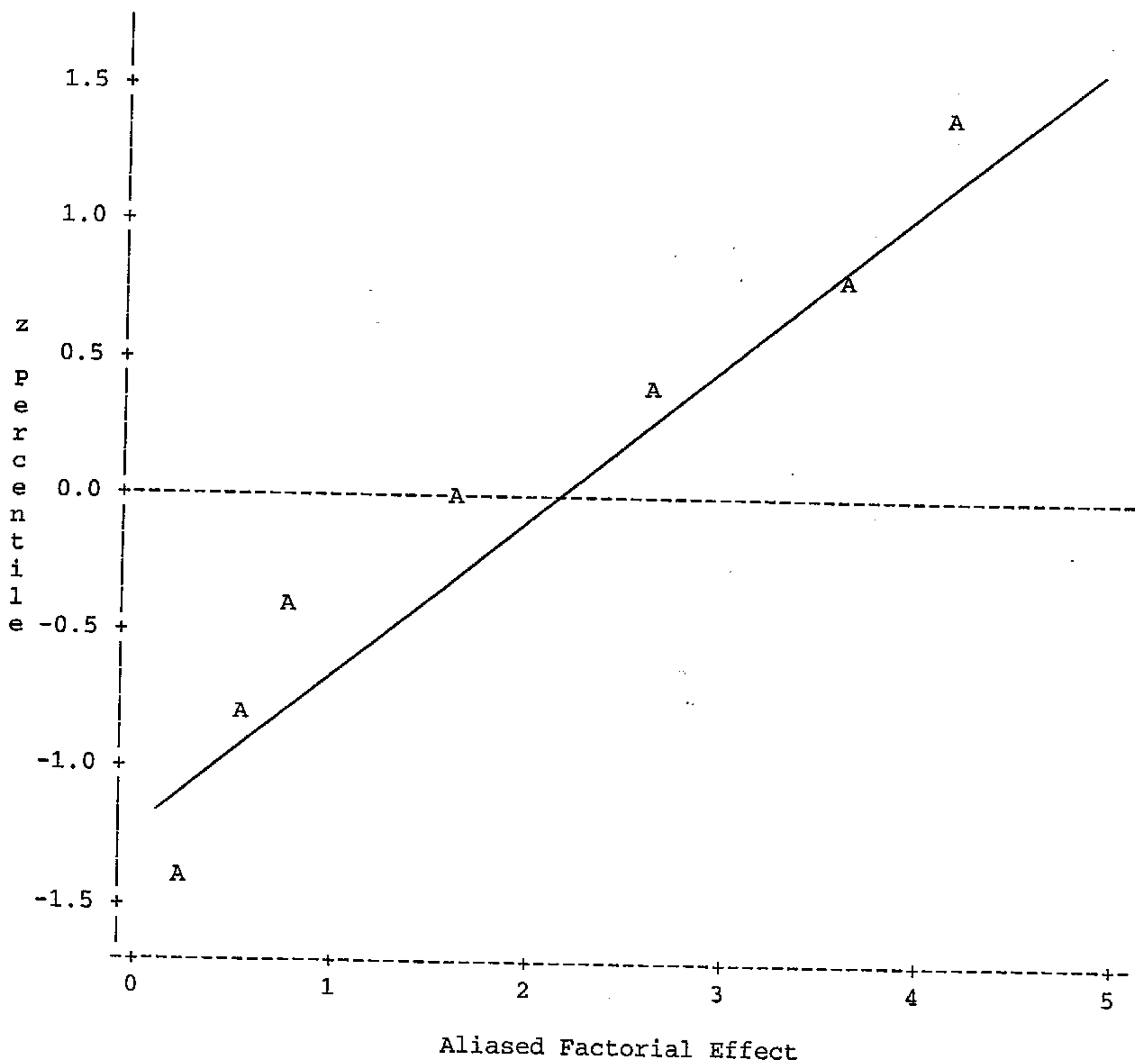
```
*The following code analyzes the second half of Fold-over Design 1.*;
*The design generators for this half are -ABD, -ACE, and -BCF.*;

Data Example2;

 Do Plyslf_m = 1 to -1 by -2; Do Reflux = 1 to -1 by -2;

 Do Plyslf_i = 1 to -1 by -2;

 Time = -Plyslf_i*Reflux; Solvent = -Plyslf_i*Plyslf_m;

 Temp = -Reflux*Plyslf_m;

     Input Strength @@; Output;

 End; End; End;

Datalines;

9.5  8.7  14.6  7.6  13.1  10.8  9.9  3.4

Proc Glm Data = Example2;

Class Plyslf_i Reflux Plyslf_m Time Solvent Temp;

Model Strength = Plyslf_i Reflux Plyslf_m Time Solvent Temp

      Plyslf_i*Temp / ss3;

Estimate 'Plyslf_i' Plyslf_i -1 1;
Estimate 'Reflux' Reflux -1 1;
Estimate 'Plyslf_m' Plyslf_m  -1 1;
Estimate 'Time' Time -1 1;
Estimate 'Solvent' Solvent -1 1;
Estimate 'Temp' Temp -1 1;

Estimate 'Plyslf_i*Temp' Plyslf_i*Temp 1 -1 -1 1 / divisor = 2;

Title 'The Second Half of Fold-over Design 1.';
```

```
*The following code analyzes the combined halves of Fold-over*;
*Design 1 where each half is treated as a separate block.*;

Data Example; Set Example; Block = -1;

Data Example2; Set Example2; Block = 1; Data Combined;

Set Example Example2;

Proc Glm Data = Combined;

     Class Block Plyslf_i Reflux Plyslf_m Time Solvent Temp;

     Model Strength = Block Plyslf_i Reflux Plyslf_m Time Solvent Temp
           Reflux*Time Plyslf_i*Time Plyslf_i*Solvent Plyslf_i*Reflux
           Plyslf_i*Plyslf_m Reflux*Plyslf_m Plyslf_i*Temp / ss3;

Estimate 'Block' Block -1 1;
Estimate 'Plyslf_i' Plyslf_i -1 1;
Estimate 'Reflux' Reflux -1 1;
Estimate 'Plyslf_m' Plyslf_m  -1 1;
Estimate 'Time' Time -1 1;
Estimate 'Solvent' Solvent -1 1;
Estimate 'Temp' Temp -1 1;

Estimate 'Reflux*Time' Reflux*Time 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Time' Plyslf_i*Time 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Solvent' Plyslf_i*Solvent 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Reflux' Plyslf_i*Reflux 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Plyslf_m' Plyslf_i*Plyslf_m 1 -1 -1 1 / divisor = 2;
Estimate 'Reflux*Plyslf_m' Reflux*Plyslf_m 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Temp' Plyslf_i*Temp 1 -1 -1 1 / divisor = 2;

Title 'Both Halves of Fold-over Design 1 Combined.';
```

```
*The following code constructs a normal probability plot of the*;
*aliased factorial effects from the second half of Fold-over*;
*Design 1.*;

Data Example3; Input Effects @@; Datalines;

4.15  1.65  0.80  2.60  0.25  3.65  0.50

Proc Rank Data = Example3  Normal = Blom  Out = Fxset;
     Var Effects; Ranks Rankefct;

Proc Plot Data = Fxset  vpercent = 70;
     Label Rankefct = 'z Percentile'  Effects = 'Aliased Factorial Effect';
     Plot Rankefct*Effects / vref = 0;

Title1'The Second Half of Fold-over Design 1.';
Title2'Normal Probability Plot of the Aliased Factorial Effects.';
```

The Second Half of Fold-over Design 1.
Normal Probability Plot of the Aliased Factorial Effects.

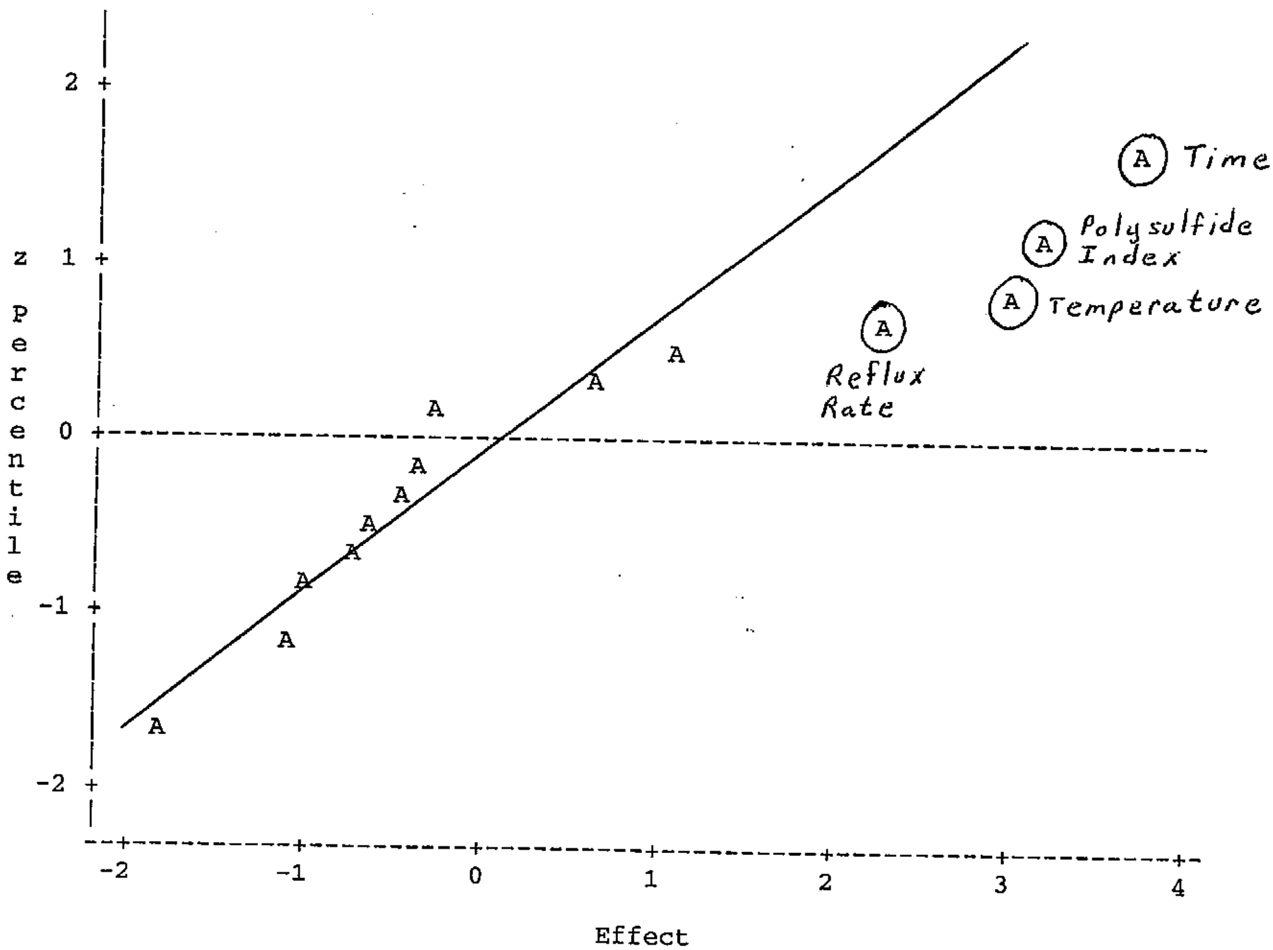
Plot of Rankefct*Effects. Legend: A = 1 obs, B = 2 obs, etc.



```
*The following code constructs a normal probability plot of the*;
*average factor effects, the aliased two-factor interaction effects,*;
*and the Block effect from the combined halves of Fold-over*;
*Design 1.*;

Data Example3; Input Effects @@; Datalines;

-1.8625  3.1875   2.2625  -0.2875   3.6875  -0.3625   2.9625
-0.9625  0.6125  -1.0875   1.0875  -0.6125  -0.6875  -0.4125

Proc Rank Data = Example3  Normal = Blom  Out = Fxset;
     Var Effects; Ranks Rankefct;

Proc Plot Data = Fxset  vpercent = 70;
     Label Rankefct = 'z Percentile'  Effects= 'Effect';
     Plot Rankefct*Effects / vref = 0;

Title1'Both Halves of Fold-over Design 1 Combined.';
Title2'Normal Probability Plot of the Average Factor Effects,';
Title3'The Aliased Two-factor Interaction Effects, and the Block Effect.';
```

Both Halves of Fold-over Design 1 Combined.
Normal Probability Plot of the Average Factor Effects,
The Aliased Two-factor Interaction Effects, and the Block Effect.

Plot of Rankefct*Effects. Legend: A = 1 obs, B = 2 obs, etc.



Both Halves of Fold-over Design 1 Combined.

The GLM Procedure

Dependent Variable: Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	14	184.0837500	13.1488393	3.95	0.3774
Error	1	3.3306250	3.3306250		
Corrected Total	15	187.4143750			

R-Square	Coeff Var	Root MSE	Strength Mean
0.982229	17.16637	1.825000	10.63125

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Block	1	13.87562500	13.87562500	4.17	0.2900
Plyslf_i	1	40.64062500	40.64062500	12.20	0.1775
Reflux	1	20.47562500	20.47562500	6.15	0.2441
Plyslf_m	1	0.33062500	0.33062500	0.10	0.8057
Time	1	54.39062500	54.39062500	16.33	0.1544
Solvent	1	0.52562500	0.52562500	0.16	0.7593
Temp	1	35.10562500	35.10562500	10.54	0.1902
Reflux*Time	1	3.70562500	3.70562500	1.11	0.4830
Plyslf_i*Time	1	1.50062500	1.50062500	0.45	0.6237
Plyslf_i*Solvent	1	4.73062500	4.73062500	1.42	0.4444
Plyslf_i*Reflux	1	4.73062500	4.73062500	1.42	0.4444
Plyslf_i*Plyslf_m	1	1.50062500	1.50062500	0.45	0.6237
Reflux*Plyslf_m	1	1.89062500	1.89062500	0.57	0.5889
Plyslf_i*Temp	1	0.68062500	0.68062500	0.20	0.7297

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Block	-1.86250000	0.91250000	-2.04	0.2900
Plyslf_i	3.18750000	0.91250000	3.49	0.1775
Reflux	2.26250000	0.91250000	2.48	0.2441
Plyslf_m	-0.28750000	0.91250000	-0.32	0.8057
Time	3.68750000	0.91250000	4.04	0.1544
Solvent	-0.36250000	0.91250000	-0.40	0.7593
Temp	2.96250000	0.91250000	3.25	0.1902
Reflux*Time	-0.96250000	0.91250000	-1.05	0.4830
Plyslf_i*Time	0.61250000	0.91250000	0.67	0.6237
Plyslf_i*Solvent	-1.08750000	0.91250000	-1.19	0.4444
Plyslf_i*Reflux	1.08750000	0.91250000	1.19	0.4444
Plyslf_i*Plyslf_m	-0.61250000	0.91250000	-0.67	0.6237
Reflux*Plyslf_m	-0.68750000	0.91250000	-0.75	0.5889
Plyslf_i*Temp	-0.41250000	0.91250000	-0.45	0.7297

The Data from the Combined Halves of Fold-over Design 1 Projected Into an Unreplicated, Blocked 2^4 Factorial Design Involving the Factors Polysulfide Index, Reflux Rate, Time, and Temperature. Only the Polysulfide Index X Reflux Rate and Reflux Rate X Time True Interaction Effects Are Not Assumed to Be Zero.

The GLM Procedure

Dependent Variable: Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	172.9243750	24.7034821	13.64	0.0007
Error	8	14.4900000	1.8112500		
Corrected Total	15	187.4143750			

R-Square	Coeff Var	Root MSE	Strength Mean
0.922685	12.65916	1.345827	10.63125

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Block	1	13.87562500	13.87562500	7.66	0.0244
Plyslf_i	1	40.64062500	40.64062500	22.44	0.0015
Reflux	1	20.47562500	20.47562500	11.30	0.0099
Time	1	54.39062500	54.39062500	30.03	0.0006
Temp	1	35.10562500	35.10562500	19.38	0.0023
Plyslf_i*Reflux	1	4.73062500	4.73062500	2.61	0.1447
Reflux*Time	1	3.70562500	3.70562500	2.05	0.1905

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Block	-1.86250000	0.67291344	-2.77	0.0244
Plyslf_i	3.18750000	0.67291344	4.74	0.0015
Reflux	2.26250000	0.67291344	3.36	0.0099
Time	3.68750000	0.67291344	5.48	0.0006
Temp	2.96250000	0.67291344	4.40	0.0023
Plyslf_i*Reflux	1.08750000	0.67291344	1.62	0.1447
Reflux*Time	-0.96250000	0.67291344	-1.43	0.1905

```
*The following code projects the data from the combined halves of*;
*Fold-over Design 1 into an unreplicated, blocked 2^4 factorial*;
*design involving the factors Polysulfide Index, Reflux Rate, Time,*;
*and Temperature.  The Polysulfide Index X Reflux Rate and*;
*Reflux Rate X Time true interaction effects are the only true*;
*interaction effects that are not assumed to be zero.  Each half of*;
*Fold-over Design 1 is treated as a separate block.*;

Data Example;

 Do Temp = -1 to 1 by 2; Do Time = -1 to 1 by 2;

  Do Reflux = -1 to 1 by 2; Do Plyslf_i = -1 to 1 by 2;

   Input Strength Block; Output;

 End; End; End; End;

Datalines;

3.4    1
7.2   -1
7.9   -1
9.5    1
9.4   -1
9.9    1
8.7    1
17.2  -1
7.6    1
10.5  -1
11.1  -1
13.1   1
13.4  -1
14.6   1
10.8   1
15.8  -1
;

Proc Glm Data = Example;

Class Block Plyslf_i Reflux Time Temp;

Model Strength = Block Plyslf_i Reflux Time Temp Plyslf_i*Reflux

                 Reflux*Time / ss3;

Estimate 'Block' Block -1 1;
Estimate 'Plyslf_i' Plyslf_i -1 1;
Estimate 'Reflux' Reflux -1 1;
Estimate 'Time' Time -1 1;
Estimate 'Temp' Temp -1 1;

Estimate 'Plyslf_i*Reflux' Plyslf_i*Reflux 1 -1 -1 1 / divisor = 2;
Estimate 'Reflux*Time' Reflux*Time 1 -1 -1 1 / divisor = 2;

Title1'The Data from the Combined Halves of Fold-over Design 1 Projected';
Title2'Into an Unreplicated, Blocked 2^4 Factorial Design Involving the';
Title3'Factors Polysulfide Index, Reflux Rate, Time, and Temperature.';
Title4'Only the Polysulfide Index X Reflux Rate and Reflux Rate X Time';
Title5'True Interaction Effects Are Not Assumed to Be Zero.';
```

All 1/8 Fractions of a 2^6 Factorial Design
Using +-BCD, +-ABE, and +-ACF as Design Generators.

------------------------------ FRACTION=1 ------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
1	-1	-1	1	-1	-1	1	1	1
-1	1	1	1	-1	-1	1	1	1
1	1	-1	-1	1	-1	1	1	1
-1	-1	1	-1	1	-1	1	1	1
-1	1	-1	-1	-1	1	1	1	1
1	-1	1	-1	-1	1	1	1	1
-1	-1	-1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1

------------------------------ FRACTION=2 ------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
1	-1	-1	-1	-1	-1	-1	1	1
-1	1	1	-1	-1	-1	-1	1	1
1	1	-1	1	1	-1	-1	1	1
-1	-1	1	1	1	-1	-1	1	1
-1	1	-1	1	-1	1	-1	1	1
1	-1	1	1	-1	1	-1	1	1
-1	-1	-1	-1	1	1	-1	1	1
1	1	1	-1	1	1	-1	1	1

------------------------------ FRACTION=3 ------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
1	1	-1	-1	-1	-1	1	-1	1
-1	-1	1	-1	-1	-1	1	-1	1
1	-1	-1	1	1	-1	1	-1	1
-1	1	1	1	1	-1	1	-1	1
-1	-1	-1	1	-1	1	1	-1	1
1	1	1	1	-1	1	1	-1	1
-1	1	-1	-1	1	1	1	-1	1
1	-1	1	-1	1	1	1	-1	1

------------------------------ FRACTION=4 ------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
-1	1	-1	-1	-1	-1	1	1	-1
1	-1	1	-1	-1	-1	1	1	-1
-1	-1	-1	1	1	-1	1	1	-1
1	1	1	1	1	-1	1	1	-1
1	-1	-1	1	-1	1	1	1	-1
-1	1	1	1	-1	1	1	1	-1
1	1	-1	-1	1	1	1	1	-1
-1	-1	1	-1	1	1	1	1	-1

```
*The following code generates all 1/8 fractions of a 2^6 factorial*;
*design using +-BCD, +-ABE, and +-ACF as design generators.*;

Data Example;

Do F = -1 to 1 by 2;
Do E = -1 to 1 by 2;
Do D = -1 to 1 by 2;
Do C = -1 to 1 by 2;
Do B = -1 to 1 by 2;
Do A = -1 to 1 by 2;

DR1 = B*C*D; DR2 = A*B*E; DR3 = A*C*F;

IF  DR1 =  1  And  DR2 =  1  And  DR3 =  1  Then  FRACTION = 1;
IF  DR1 = -1  And  DR2 =  1  And  DR3 =  1  Then  FRACTION = 2;
IF  DR1 =  1  And  DR2 = -1  And  DR3 =  1  Then  FRACTION = 3;
IF  DR1 =  1  And  DR2 =  1  And  DR3 = -1  Then  FRACTION = 4;
IF  DR1 = -1  And  DR2 = -1  And  DR3 =  1  Then  FRACTION = 5;
IF  DR1 = -1  And  DR2 =  1  And  DR3 = -1  Then  FRACTION = 6;
IF  DR1 =  1  And  DR2 = -1  And  DR3 = -1  Then  FRACTION = 7;
IF  DR1 = -1  And  DR2 = -1  And  DR3 = -1  Then  FRACTION = 8; Output;

End; End; End; End; End; End;

Proc Sort Data = Example; By FRACTION;
Proc Print Data = Example; By FRACTION;
     ID A B C D E F; Var DR1 DR2 DR3;

Title1'All 1/8 Fractions of a 2^6 Factorial Design';
Title2'Using +-BCD, +-ABE, and +-ACF as Design Generators.';
```

```
*The following code analyzes the first half of Fold-over Design 2.*;
*The design generators for this half are BCD, ABE, and ACF.*;

Data Example;

 Do Plyslf_m = -1 to 1 by 2; Do Reflux = -1 to 1 by 2;

 Do Plyslf_i = -1 to 1 by 2;

 Time = Reflux*Plyslf_m; Solvent = Plyslf_i*Reflux;

 Temp = Plyslf_i*Plyslf_m;

    Input Strength @@; Output;

 End; End; End;

Datalines;

13.4  10.5  11.3  10.7  7.2  12.5  9.5  15.8

Proc Glm Data = Example;

Class Plyslf_i Reflux Plyslf_m Time Solvent Temp;

Model Strength = Plyslf_i Reflux Plyslf_m Time Solvent Temp

      Plyslf_i*Time / ss3;

Estimate 'Plyslf_i' Plyslf_i -1 1;
Estimate 'Reflux' Reflux -1 1;
Estimate 'Plyslf_m' Plyslf_m  -1 1;
Estimate 'Time' Time -1 1;
Estimate 'Solvent' Solvent -1 1;
Estimate 'Temp' Temp -1 1;

Estimate 'Plyslf_i*Time' Plyslf_i*Time 1 -1 -1 1 / divisor = 2;

Title'The First Half of Fold-over Design 2.';
```

------------------------------ FRACTION=5 ------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
1	1	-1	1	-1	-1	-1	-1	1
-1	-1	1	1	-1	-1	-1	-1	1
1	-1	-1	-1	1	-1	-1	-1	1
-1	1	1	-1	1	-1	-1	-1	1
-1	-1	-1	-1	-1	1	-1	-1	1
1	1	1	-1	-1	1	-1	-1	1
-1	1	-1	1	1	1	-1	-1	1
1	-1	1	1	1	1	-1	-1	1

------------------------------ FRACTION=6 ------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
-1	1	-1	1	-1	-1	-1	1	-1
1	-1	1	1	-1	-1	-1	1	-1
-1	-1	-1	-1	1	-1	-1	1	-1
1	1	1	-1	1	-1	-1	1	-1
1	-1	-1	-1	-1	1	-1	1	-1
-1	1	1	-1	-1	1	-1	1	-1
1	1	-1	1	1	1	-1	1	-1
-1	-1	1	1	1	1	-1	1	-1

------------------------------ FRACTION=7 ------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
-1	-1	-1	1	-1	-1	1	-1	-1
1	1	1	1	-1	-1	1	-1	-1
-1	1	-1	-1	1	-1	1	-1	-1
1	-1	1	-1	1	-1	1	-1	-1
1	1	-1	-1	-1	1	1	-1	-1
-1	-1	1	-1	-1	1	1	-1	-1
1	-1	-1	1	1	1	1	-1	-1
-1	1	1	1	1	1	1	-1	-1

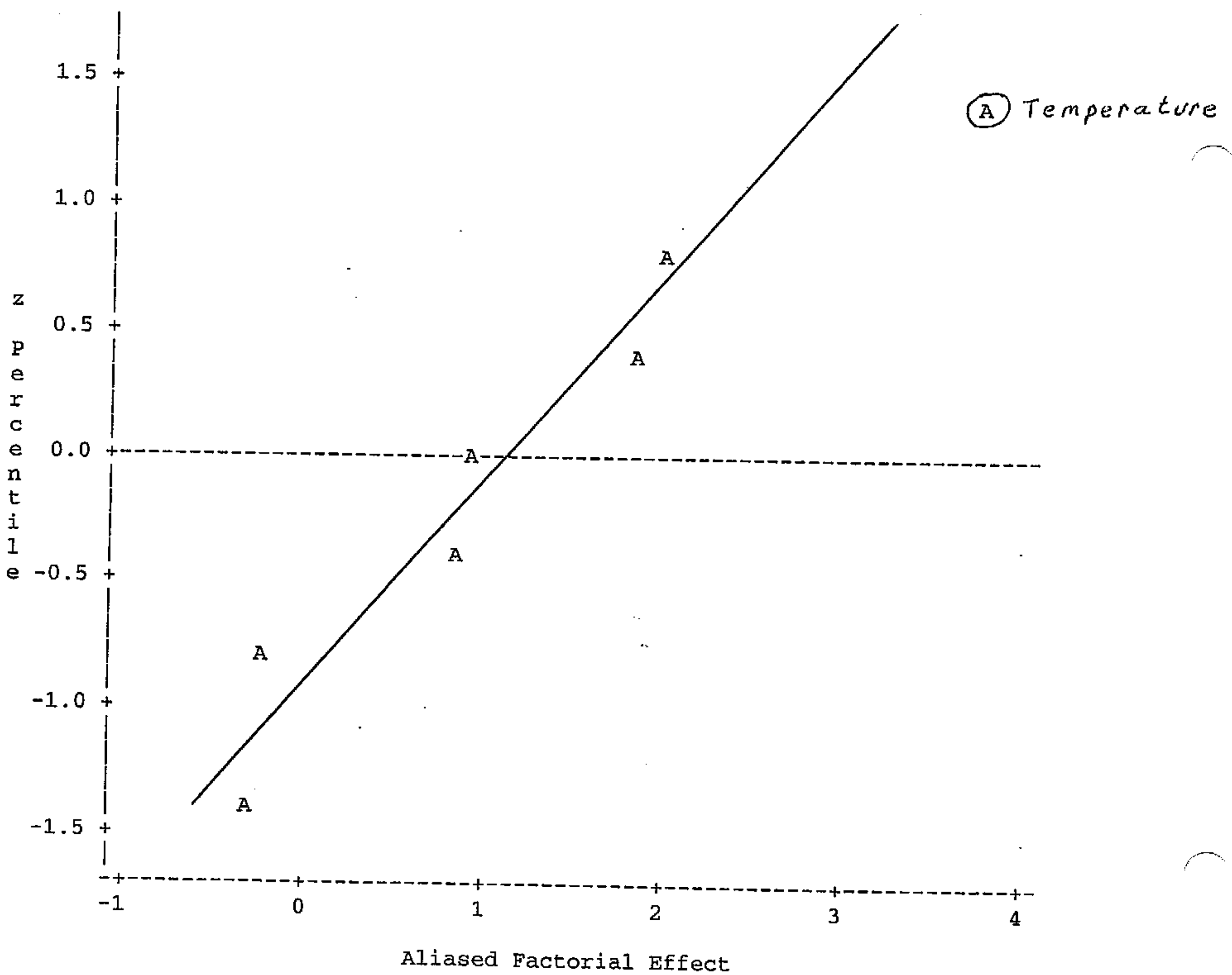
------------------------------ FRACTION=8 ------------------------------

A	B	C	D	E	F	DR1	DR2	DR3
-1	-1	-1	-1	-1	-1	-1	-1	-1
1	1	1	-1	-1	-1	-1	-1	-1
-1	1	-1	1	1	-1	-1	-1	-1
1	-1	1	1	1	-1	-1	-1	-1
1	1	-1	1	-1	1	-1	-1	-1
-1	-1	1	1	-1	1	-1	-1	-1
1	-1	-1	-1	1	1	-1	-1	-1
-1	1	1	-1	1	1	-1	-1	-1

```
*The following code constructs a normal probability plot of the*;
*aliased factorial effects from the first half of Fold-over*;
*Design 2.*;

Data Example3; Input Effects @@; Datalines;

2.025  0.925  -0.225  1.875  0.825  3.775  -0.325

Proc Rank Data = Example3  Normal = Blom  Out = Fxset;
     Var Effects; Ranks Rankefct;

Proc Plot Data = Fxset  vpercent = 70;
     Label Rankefct = 'z Percentile'  Effects = 'Aliased Factorial Effect';
     Plot Rankefct*Effects / vref = 0;

Title1'The First Half of Fold-over Design 2.';
Title2'Normal Probability Plot of the Aliased Factorial Effects.';
```

The First Half of Fold-over Design 2.
Normal Probability Plot of the Aliased Factorial Effects.

Plot of Rankefct*Effects. Legend: A = 1 obs, B = 2 obs, etc.



The First Half of Fold-over Design 2.

The GLM Procedure

Dependent Variable: Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	47.11875000	6.73125000	.	.
Error	0	0.00000000	.		
Corrected Total	7	47.11875000			

R-Square	Coeff Var	Root MSE	Strength Mean
1.000000	.	.	11.36250

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Plyslf_i	1	8.20125000	8.20125000	.	.
Reflux	1	1.71125000	1.71125000	.	.
Plyslf_m	1	0.10125000	0.10125000	.	.
Time	1	7.03125000	7.03125000	.	.
Solvent	1	1.36125000	1.36125000	.	.
Temp	1	28.50125000	28.50125000	.	.
Plyslf_i*Time	1	0.21125000	0.21125000	.	.

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Plyslf_i	2.02500000	.	.	.
Reflux	0.92500000	.	.	.
Plyslf_m	-0.22500000	.	.	.
Time	1.87500000	.	.	.
Solvent	0.82500000	.	.	.
Temp	3.77500000	.	.	.
Plyslf_i*Time	-0.32500000	.	.	.

The Second Half of Fold-over Design 2.

The GLM Procedure

Dependent Variable: Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	135.9200000	19.4171429	.	.
Error	0	0.0000000	.		
Corrected Total	7	135.9200000			

R-Square	Coeff Var	Root MSE	Strength Mean
1.000000	.	.	11.30000

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Plyslf_i	1	40.50000000	40.50000000	.	.
Reflux	1	0.18000000	0.18000000	.	.
Plyslf_m	1	11.52000000	11.52000000	.	.
Time	1	32.00000000	32.00000000	.	.
Solvent	1	1.28000000	1.28000000	.	.
Temp	1	48.02000000	48.02000000	.	.
Plyslf_i*Time	1	2.42000000	2.42000000	.	.

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Plyslf_i	4.50000000	.	.	.
Reflux	0.30000000	.	.	.
Plyslf_m	2.40000000	.	.	.
Time	4.00000000	.	.	.
Solvent	0.80000000	.	.	.
Temp	4.90000000	.	.	.
Plyslf_i*Time	1.10000000	.	.	.

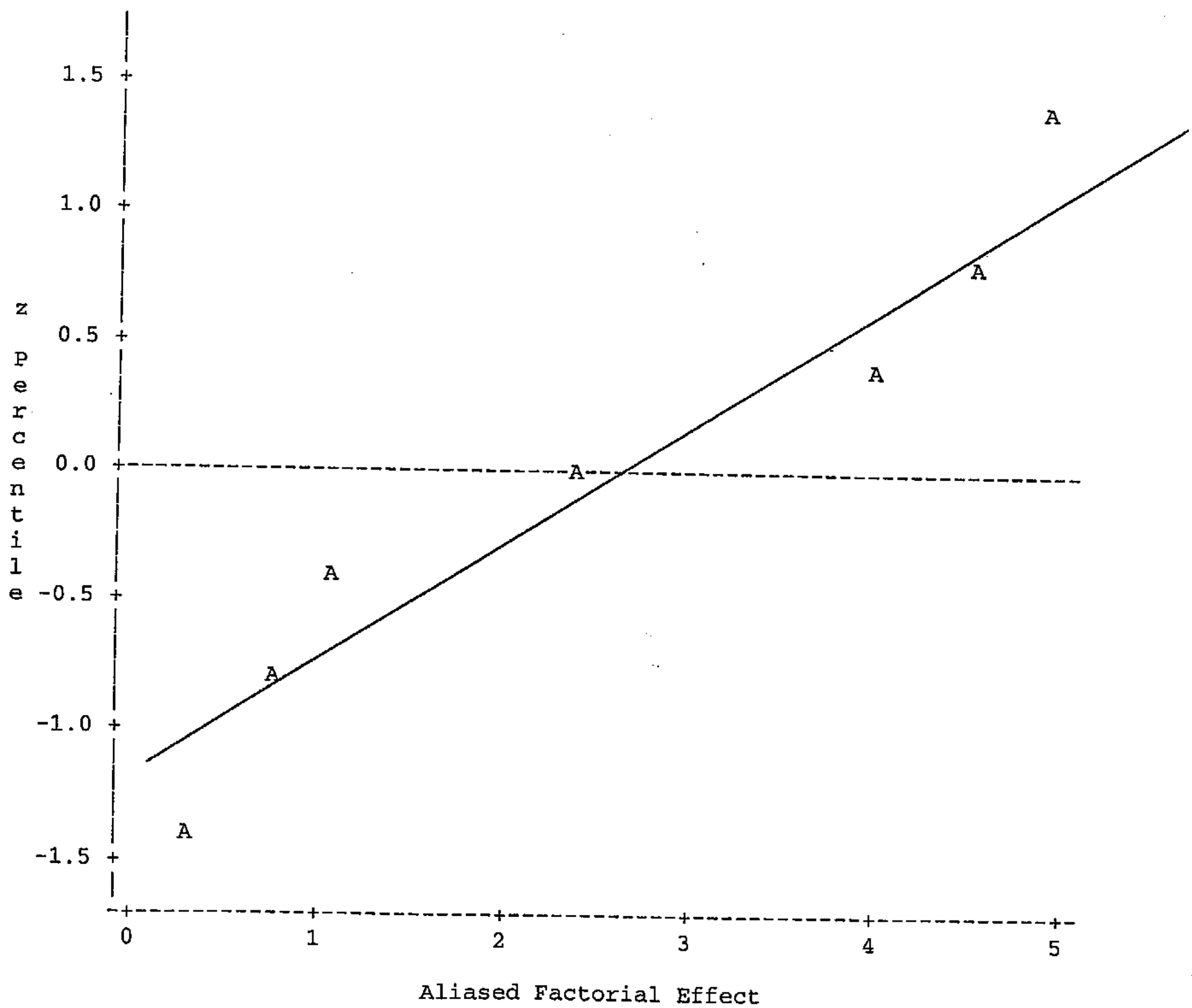
```
*The following code analyzes the second half of Fold-over Design 2.*;
*The design generators for this half are -BCD, -ABE, and -ACF.*;

Data Example2;

 Do Plyslf_m = 1 to -1 by -2; Do Reflux = 1 to -1 by -2;

 Do Plyslf_i = 1 to -1 by -2;

 Time = -Reflux*Plyslf_m; Solvent = -Plyslf_i*Reflux;

 Temp = -Plyslf_i*Plyslf_m;

     Input Strength @@; Output;

 End; End; End;

Datalines;

9.5  11.8  15.1  13.6  17.1  7.4  12.5  3.4

Proc Glm Data = Example2;

Class Plyslf_i Reflux Plyslf_m Time Solvent Temp;

Model Strength = Plyslf_i Reflux Plyslf_m Time Solvent Temp

      Plyslf_i*Time / ss3;

Estimate 'Plyslf_i' Plyslf_i -1 1;
Estimate 'Reflux' Reflux -1 1;
Estimate 'Plyslf_m' Plyslf_m  -1 1;
Estimate 'Time' Time -1 1;
Estimate 'Solvent' Solvent -1 1;
Estimate 'Temp' Temp -1 1;

Estimate 'Plyslf_i*Time' Plyslf_i*Time 1 -1 -1 1 / divisor = 2;

Title 'The Second Half of Fold-over Design 2.';
```

```
*The following code analyzes the combined halves of Fold-over*;
*Design 2 where each half is treated as a separate block.*;

Data Example; Set Example; Block = -1;

Data Example2; Set Example2; Block = 1; Data Combined;

Set Example Example2;

Proc Glm Data = Combined;

     Class Block Plyslf_i Reflux Plyslf_m Time Solvent Temp;

     Model Strength = Block Plyslf_i Reflux Plyslf_m Time Solvent Temp
          Reflux*Solvent Plyslf_i*Solvent Plyslf_i*Temp Reflux*Plyslf_m
          Plyslf_i*Reflux Plyslf_i*Plyslf_m Plyslf_i*Time / ss3;

Estimate 'Block' Block -1 1;
Estimate 'Plyslf_i' Plyslf_i -1 1;
Estimate 'Reflux' Reflux -1 1;
Estimate 'Plyslf_m' Plyslf_m  -1 1;
Estimate 'Time' Time -1 1;
Estimate 'Solvent' Solvent -1 1;
Estimate 'Temp' Temp -1 1;

Estimate 'Reflux*Solvent' Reflux*Solvent 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Solvent' Plyslf_i*Solvent 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Temp' Plyslf_i*Temp 1 -1 -1 1 / divisor = 2;
Estimate 'Reflux*Plyslf_m' Reflux*Plyslf_m 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Reflux' Plyslf_i*Reflux 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Plyslf_m' Plyslf_i*Plyslf_m 1 -1 -1 1 / divisor = 2;
Estimate 'Plyslf_i*Time' Plyslf_i*Time 1 -1 -1 1 / divisor = 2;

Title 'Both Halves of Fold-over Design 2 Combined.';
```

```
*The following code constructs a normal probability plot of the*;
*aliased factorial effects from the second half of Fold-over*;
*Design 2.*;

Data Example3; Input Effects @@; Datalines;

4.5  0.3  2.4  4.0  0.8  4.9  1.1

Proc Rank Data = Example3  Normal = Blom  Out = Fxset;
     Var Effects; Ranks Rankefct;

Proc Plot Data = Fxset  vpercent = 70;
     Label Rankefct = 'z Percentile'  Effects = 'Aliased Factorial Effect';
     Plot Rankefct*Effects / vref = 0;

Title1'The Second Half of Fold-over Design 2.';
Title2'Normal Probability Plot of the Aliased Factorial Effects.';
```

The Second Half of Fold-over Design 2.
Normal Probability Plot of the Aliased Factorial Effects.

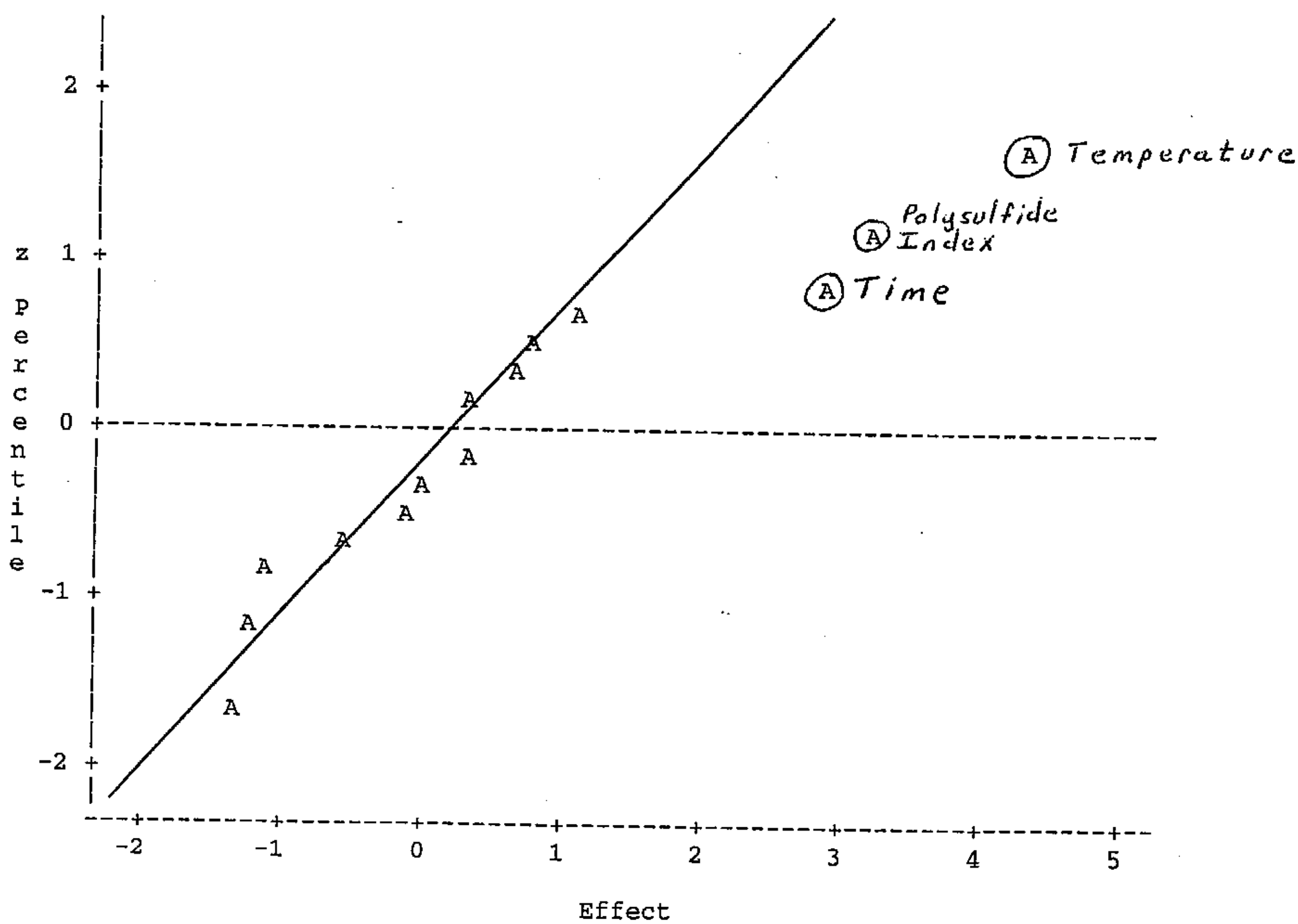
Plot of Rankefct*Effects. Legend: A = 1 obs, B = 2 obs, etc.



```
*The following code constructs a normal probability plot of the*;
*average factor effects, the aliased two-factor interaction effects,*;
*and the Block effect from the combined halves of Fold-over*;
*Design 2.*;

Data Example3; Input Effects @@; Datalines;

-0.0625  3.2625   0.6125   1.0875  2.9375   0.8125  4.3375
-1.2375  0.3125  -1.3125  -1.0625  0.0125  -0.5625  0.3875

Proc Rank Data = Example3  Normal = Blom  Out = Fxset;
     Var Effects; Ranks Rankefct;

Proc Plot Data = Fxset  vpercent = 70;
     Label Rankefct = 'z Percentile'  Effects = 'Effect';
     Plot Rankefct*Effects / vref = 0;

Title1'Both Halves of Fold-over Design 2 Combined.';
Title2'Normal Probability Plot of the Average Factor Effects,';
Title3'The Aliased Two-factor Interaction Effects, and the Block Effect.';
```

Both Halves of Fold-over Design 2 Combined.
Normal Probability Plot of the Average Factor Effects,
The Aliased Two-factor Interaction Effects, and the Block Effect.

Plot of Rankefct*Effects. Legend: A = 1 obs, B = 2 obs, etc.



Both Halves of Fold-over Design 2 Combined.

The GLM Procedure

Dependent Variable: Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	14	181.0237500	12.9302679	6.37	0.3021
Error	1	2.0306250	2.0306250		
Corrected Total	15	183.0543750			

R-Square	Coeff Var	Root MSE	Strength Mean
0.988907	12.57584	1.425000	11.33125

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Block	1	0.01562500	0.01562500	0.01	0.9443
Plyslf_i	1	42.57562500	42.57562500	20.97	0.1369
Reflux	1	1.50062500	1.50062500	0.74	0.5480
Plyslf_m	1	4.73062500	4.73062500	2.33	0.3692
Time	1	34.51562500	34.51562500	17.00	0.1515
Solvent	1	2.64062500	2.64062500	1.30	0.4583
Temp	1	75.25562500	75.25562500	37.06	0.1036
Reflux*Solvent	1	6.12562500	6.12562500	3.02	0.3326
Plyslf_i*Solvent	1	0.39062500	0.39062500	0.19	0.7369
Plyslf_i*Temp	1	6.89062500	6.89062500	3.39	0.3166
Reflux*Plyslf_m	1	4.51562500	4.51562500	2.22	0.3761
Plyslf_i*Reflux	1	0.00062500	0.00062500	0.00	0.9888
Plyslf_i*Plyslf_m	1	1.26562500	1.26562500	0.62	0.5746
Plyslf_i*Time	1	0.60062500	0.60062500	0.30	0.6829

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Block	-0.06250000	0.71250000	-0.09	0.9443
Plyslf_i	3.26250000	0.71250000	4.58	0.1369
Reflux	0.61250000	0.71250000	0.86	0.5480
Plyslf_m	1.08750000	0.71250000	1.53	0.3692
Time	2.93750000	0.71250000	4.12	0.1515
Solvent	0.81250000	0.71250000	1.14	0.4583
Temp	4.33750000	0.71250000	6.09	0.1036
Reflux*Solvent	-1.23750000	0.71250000	-1.74	0.3326
Plyslf_i*Solvent	0.31250000	0.71250000	0.44	0.7369
Plyslf_i*Temp	-1.31250000	0.71250000	-1.84	0.3166
Reflux*Plyslf_m	-1.06250000	0.71250000	-1.49	0.3761
Plyslf_i*Reflux	0.01250000	0.71250000	0.02	0.9888
Plyslf_i*Plyslf_m	-0.56250000	0.71250000	-0.79	0.5746
Plyslf_i*Time	0.38750000	0.71250000	0.54	0.6829

The Data from the Combined Halves of Fold-over Design 2 Projected
Into a Twice-replicated, Blocked 2^3 Factorial Design Involving
The Factors Polysulfide Index, Time, and Temperature.
All True Interaction Effects Are Assumed to Be Zero.

The GLM Procedure

Dependent Variable: Strength

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	152.3625000	38.0906250	13.65	0.0003
Error	11	30.6918750	2.7901705		
Corrected Total	15	183.0543750			

R-Square	Coeff Var	Root MSE	Strength Mean
0.832335	14.74136	1.670380	11.33125

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Block	1	0.01562500	0.01562500	0.01	0.9417
Plyslf_i	1	42.57562500	42.57562500	15.26	0.0024
Time	1	34.51562500	34.51562500	12.37	0.0048
Temp	1	75.25562500	75.25562500	26.97	0.0003

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Block	-0.06250000	0.83519017	-0.07	0.9417
Plyslf_i	3.26250000	0.83519017	3.91	0.0024
Time	2.93750000	0.83519017	3.52	0.0048
Temp	4.33750000	0.83519017	5.19	0.0003

```
*The following code projects the data from the combined halves of*;
*Fold-over Design 2 into a twice-replicated, blocked 2^3 factorial*;
*design involving the factors Polysulfide Index, Time, and Temperature.*;
*All true interaction effects are assumed to be zero and each half*;
*of Fold-over Design 2 is treated as a separate block.*;

Data Example;

 Do Block = -1 to 1 by 2; Do Temp = -1 to 1 by 2;

  Do Time = -1 to 1 by 2; Do Plyslf_i = -1 to 1 by 2;

   Input Strength @@; Output;

 End; End; End; End;

Datalines;

7.2  10.7  9.5  10.5  11.3  12.5  13.4  15.8
3.4   9.5  7.4  15.1  11.8  12.5  13.6  17.1

Proc Glm Data = Example;

Class Block Plyslf_i Time Temp;

Model Strength = Block Plyslf_i Time Temp / ss3;

Estimate 'Block' Block -1 1;
Estimate 'Plyslf_i' Plyslf_i -1 1;
Estimate 'Time' Time -1 1;
Estimate 'Temp' Temp -1 1;


Title1'The Data from the Combined Halves of Fold-over Design 2 Projected';
Title2'Into a Twice-replicated, Blocked 2^3 Factorial Design Involving';
Title3'The Factors Polysulfide Index, Time, and Temperature.';
Title4'All True Interaction Effects Are Assumed to Be Zero.';

Run;
```