Math 172 | Quiz 10 | Name: ______________

4 Nov 2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. Determine if the following converge conditionally, converge absolutely, or diverge.

   (a) $\boxed{3}$ $\sum \frac{\cos n}{n^2+n+2}$

Since $0 < \left| \frac{\cos n}{n^2+n+2} \right| < \frac{1}{n^2}$ & $\sum \frac{1}{n^2}$ converges $(p=2>1)$, by comparison $\sum \left| \frac{\cos n}{n^2+n+2} \right|$ also converges, i.e. $\sum \frac{\cos n}{n^2+n+2}$ converges absolutely.

   (b) $\boxed{4}$ $\sum_{n=3}^{\infty} \frac{n}{n^2-1}(-1)^{n+1}$

Since $0 < \frac{1}{n} < \left| \frac{n(-1)^{n+1}}{n^2-1} \right|$ & $\sum_{n=3}^{\infty} \frac{1}{n}$ diverges, by comparison, $\sum_{n=3}^{\infty} \left| \frac{n(-1)^{n+1}}{n^2-1} \right|$ also diverges, i.e. $\sum_{n=3}^{\infty} \frac{n(-1)^{n+1}}{n^2-1}$ does not converge absolutely.

To check for conditional convergence we apply the Alternating Series Test, since $\frac{n}{n^2-1} > 0$, $\frac{n}{n^2-1} > \frac{n+1}{(n+1)^2-1}$, and $\frac{n}{n^2-1} \xrightarrow[n\to\infty]{} 0$, $\sum_{n=3}^{\infty} \frac{n}{n^2-1}(-1)^{n+1}$ converges & hence converges conditionally.

2. Use the Integral Test to show the following converges. You do not need to verify the hypotheses of the test.

$$\sum_{n=4}^{\infty} \frac{1}{n(\ln n)^5}$$

Let $f(x) = \frac{1}{x(\ln x)^5}$. Assume $f(x)$ satisfies the hypotheses of the Integral Test (it does), then we consider $\int_4^{\infty} \frac{1}{x(\ln x)^5}\,dx = \int_{\ln 4}^{\infty} \frac{1}{u^5}\,du$ which is a convergent p-integral $(p=5>1)$,

$u = \ln x$ $\quad 4 \longrightarrow \ln 4$

$du = \frac{1}{x}dx$ $\quad x \to \infty \longmapsto u \longrightarrow \infty$

so by the Integral Test, $\sum_{n=4}^{\infty} \frac{1}{n(\ln n)^5}$ converges as well.