1. Determine if the following converge conditionally, converge absolutely, or diverge.

(a) $\sum_{n=3}^{\infty} \frac{\cos n}{n^2 + n + 2}$

Since $0 < \left| \frac{\cos n}{n^2 + n + 2} \right| < \frac{1}{n^2}$, by the Comparison Test, $\sum_{n=3}^{\infty} \frac{1}{n^2}$ converges (p-series), by comparison.

$\sum_{n=3}^{\infty} \left| \frac{\cos n}{n^2 + n + 2} \right|$ also converges, i.e. $\sum_{n=3}^{\infty} \frac{\cos n}{n^2 + n + 2}$ converges absolutely.

(b) $\sum_{n=3}^{\infty} \frac{n}{n^2 - 1} (-1)^{n+1}$

Since $0 < \frac{n}{n^2 - 1} < \frac{1}{n}$, $\sum_{n=3}^{\infty} \frac{1}{n}$ diverges, by comparison.

$\sum_{n=3}^{\infty} \frac{n}{n^2 - 1} (-1)^{n+1}$ also diverges, i.e. $\sum_{n=3}^{\infty} \frac{n}{n^2 - 1} (-1)^{n+1}$ does not converge absolutely.

To check for conditional convergence, we apply the Alternating Series Test.

Since $\frac{n}{n^2 - 1} > 0$, $\frac{n}{(n+1)^2 - 1} > \frac{n+1}{n^2 - 1}$, and $\frac{n}{n^2 - 1} \to 0$ as $n \to \infty$, $\sum_{n=3}^{\infty} \frac{n}{n^2 - 1} (-1)^{n+1}$ converges and hence converges conditionally.

2. Use the Integral Test to show the following converges. You do not need to verify the hypotheses of the test.

$$\sum_{n=4}^{\infty} \frac{1}{n \ln(n)^5}$$

Let $f(x) = \frac{1}{x (\ln x)^5}$. Assume $f(x)$ satisfies the hypotheses of the Integral Test (it does).

Then we consider

$$\int_{4}^{\infty} \frac{1}{x (\ln x)^5} \, dx = \int_{4}^{\infty} \frac{1}{u^5} \, du$$

which is a convergent p-integral ($p > 1$).

$$u = \ln x \quad 4 \to 4 \quad dx = \frac{1}{u} \, du \quad x \to 1 \quad u \to -\infty$$

So by the Integral Test, $\sum_{n=4}^{\infty} \frac{1}{n \ln(n)^5}$ converges as well.