Math 172 | Quiz 11 | Name: ____________
9 Nov 2016 | Show appropriate work | Point Values in $\boxed{\text{boxes}}$.

Provide appropriate arguments for each of the following.

1. $\boxed{6}$ Find the Radius of Convergence for each of the following.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n(x-3)^n}{4^n}$ Apply Root Test $\displaystyle\sqrt[n]{\left|\frac{n(x-3)^n}{4^n}\right|} = \sqrt[n]{n}\,\frac{|x-3|}{4} \xrightarrow[n\to\infty]{} \frac{|x-3|}{4} < 1$

so $|x-3| < 4$

i.e. $R = 4$

(b) $\displaystyle\sum_{n=0}^{\infty} x^n \left(\frac{n+2}{n}\right)^{n^2}$ Apply Root Test $\displaystyle\sqrt[n]{\left|x^n\left(\frac{n+2}{n}\right)^{n^2}\right|} = |x| \left(\frac{n+2}{n}\right)^n$

$\displaystyle = |x| \left(1+\frac{2}{n}\right)^n \xrightarrow[n\to\infty]{} |x|e^2 < 1$

so $|x| < \frac{1}{e^2}$ i.e. $R = \frac{1}{e^2}$

2. $\boxed{4}$ Determine if the following converges or diverges.

$$\sum_{n=2}^{\infty} \frac{3n+7}{\sqrt{2n^6 - n^4 - 1}}$$

Idea: $\sim \sum \frac{n}{n^3} = \sum \frac{1}{n^2}$

Argument

Since $\frac{3n+7}{\sqrt{2n^6-n^4-1}}$ & $\frac{1}{n^2} > 0$ we can apply the L.C.T.

To that end we compute the limit $\displaystyle\lim_{n\to\infty} \frac{3n+7}{\sqrt{2n^6-n^4-1}} \Big/ \frac{1}{n^2} = \frac{3}{\sqrt{2}}$.

Since $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$ is a convergent p-series $(p=2>1)$ & the limit is positive and finite, i.e. $0 < \frac{3}{\sqrt{2}} < \infty$, $\displaystyle\sum_{n=2}^{\infty} \frac{3n+7}{\sqrt{2n^6-n^4-1}}$ also converges.