

Provide appropriate arguments for each of the following.

1. 4 Find the Radius of Convergence for the following.

$$(a) \sum_{n=0}^{\infty} \frac{(2x-4)^n}{n^2 6^n}$$

Root Test

$$\sqrt[n]{\left| \frac{(2x-4)^n}{n^2 6^n} \right|} = \frac{|2x-4|}{\sqrt[n]{n^2} \cdot 6} \xrightarrow{n \rightarrow \infty} \frac{|2x-4|}{6} < 1$$

so

$$|2x-4| < 6$$

or

$$|x-2| < 3$$

so the Radius of Convergence is 3.

2. 6 Find the first four nonzero terms of the Taylor Series for $f(x) = (2x+1)^{3/2}$ about $c = 4$.

n	$f^{(n)}(x)$	$f^{(n)}(4)$
0	$(2x+1)^{3/2}$	27
1	$\frac{3}{2} (2x+1)^{1/2} (2)$	$3(3) = 9$
2	$3 \left(\frac{1}{2}\right) (2x+1)^{-1/2} (2)$	$3\left(\frac{1}{3}\right) = 1$
3	$3 \left(-\frac{1}{2}\right) (2x+1)^{-3/2} (2)$	$-3\left(\frac{1}{27}\right) = -\frac{1}{9}$

$$f(x) = 27 + 9(x-4) + \frac{1}{2}(x-4)^2 - \frac{1}{9}\left(\frac{1}{3!}\right)(x-4)^3 + \dots$$