Math 172 | Quiz 14 | Name: ________________

7 Dec 2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

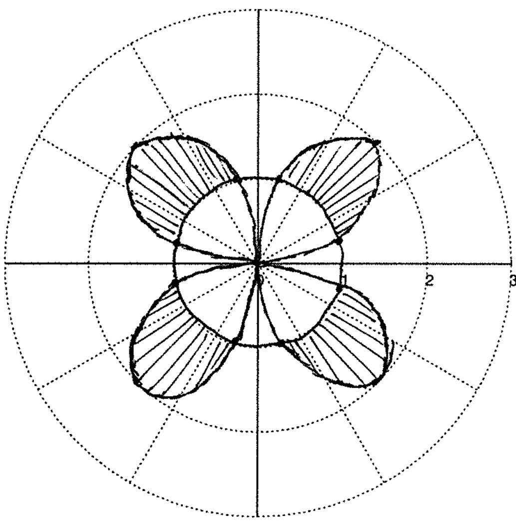
1. $\boxed{2}$ Convert the following from polar to rectangular coordinates.

$$r = \frac{2}{3\cos\theta + 4\sin\theta}$$

$$3r\cos\theta + 4r\sin\theta = 2$$

$$3x + 4y = 2$$

2. $\boxed{5}$ Carefully sketch the polar curves $r = 2\sin 2\theta$ and $r = 1$ on the grid provided and then find the area inside $r = 2\sin 2\theta$ and outside $r = 1$. You may need the indentity $\sin^2 x = (1 - \cos 2x)/2$.



$r = 2\sin 2\theta$ & $r = 1$ intersect when $\sin 2\theta = \frac{1}{2}$

so $2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ or $\cdots$ so $\theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$ or $\cdots$

$$\text{Area} = 4 \cdot \frac{1}{2}\int_{\pi/12}^{5\pi/12} \left[4\sin^2 2\theta - 1\right] d\theta = 2\int_{\pi/12}^{5\pi/12} \left[2 - 2\cos 4\theta - 1\right] d\theta$$

Symmetry

$$= 2\int_{\pi/12}^{5\pi/12} (1 - 2\cos 4\theta)\, d\theta = 2\left(\theta - \frac{1}{2}\sin 4\theta\right)\Bigg|_{\pi/12}^{5\pi/12}$$

$$= 2\left(\left[\frac{5\pi}{12} - \frac{\pi}{12}\right] - \frac{1}{2}\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)\right) = \frac{2\pi}{3} + \sqrt{3}$$

3. $\boxed{3}$ Compute the length of the curve $r = 2\theta^2$ for $\theta \in [0, \sqrt{5}]$.

$$r' = 4\theta$$

$$r^2 + (r')^2 = 4\theta^4 + 16\theta^2$$

$$s = \int_0^{\sqrt{5}} \sqrt{4\theta^4 + 16\theta^2}\, d\theta = \int_0^{\sqrt{5}} 2\theta\sqrt{\theta^2 + 4}\, d\theta = \int_4^9 u^{1/2}\, du = \frac{2}{3}u^{3/2}\Big|_4^9 = \frac{2}{3}(27 - 8)$$

$$= \frac{38}{3}$$

$u = \theta^2 + 4$

$du = 2\theta\, d\theta$