Math 172 | Quiz 4 | Name: ______________
30 Sept 2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{3}$ When using trigonometric substitution we discussed three basic forms of substitution. For integrals involving the following forms, state the appropriate substitution, i.e. $x = blah$, and the appropriate change of variables term, i.e. $dx = stuff$.

(a) $\sqrt{a^2+x^2}$ (b) $\sqrt{x^2-a^2}$ (c) $\sqrt{a^2-x^2}$

$x = a\tan\theta$ $x = a\sec\theta$ $x = a\sin\theta$

$dx = a\sec^2\theta\,d\theta$ $dx = a\sec\theta\tan\theta\,d\theta$ $dx = a\cos\theta\,d\theta$

2. Integrate.

(a) $\boxed{3}$ $\int \sin^5 x\cos^3 x\,dx = \int \sin^5 x\,(1-\sin^2 x)\cos x\,dx = \int u^5(1-u^2)\,du$

$u = \sin x \quad du = \cos x\,dx$

$= \int (u^5 - u^7)\,du = \frac{u^6}{6} - \frac{u^8}{8} + C$

$= \frac{1}{6}\sin^6 x - \frac{1}{8}\sin^8 x + C$

(b) $\boxed{4}$ $\int \sec^4 5\theta\tan^3 5\theta\,d\theta = \int (1+\tan^2 5\theta)\tan^3 5\theta\,\sec^2 5\theta\,d\theta$

$u = \tan 5\theta \quad du = 5\sec^2 5\theta\,d\theta$

$= \frac{1}{5}\int (1+u^2)u^3\,du = \frac{1}{5}\left(\frac{\tan^4 5\theta}{4} + \frac{\tan^6 5\theta}{6}\right) + C$

—— OR ——

$= \int \sec^3 5\theta\,(\sec^2 5\theta - 1)\sec 5\theta\tan 5\theta\,d\theta$

$u = \sec 5\theta \quad du = 5\sec\theta\tan\theta\,d\theta$

$= \frac{1}{5}\int u^3(u^2-1)\,du = \frac{1}{5}\left(\frac{\sec^6 5\theta}{6} - \frac{\sec^4 5\theta}{4}\right) + C$