Math 172 Quiz 5 Name: ________________

7 Oct 2016 Show Appropriate Work Point Values in $\boxed{\text{boxes}}$.

Some identities which may or may not be helpful.

(1) $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ (3) $\sin 2x = 2\ \sin x\ \cos x$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+c \qquad \int \sec^3 u\, du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + c$$

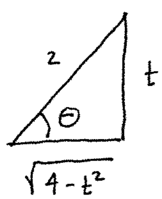
1. $\boxed{5}$ Integrate.

$$\int \sqrt{4-t^2}\, dt$$

Let $t = 2\sin\theta$ so $4-t^2 = 4-4\sin^2\theta$

$dt = 2\cos\theta\, d\theta$ $= 4\cos^2\theta$

$$\int\sqrt{4-t^2}\,dt = \int 4\cos^2\theta\, d\theta = \int 2(1+\cos 2\theta)\,d\theta \quad \longleftarrow \text{ by (1) above}$$



$$= 2\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$

$$= 2\left(\theta + \sin\theta\cos\theta\right) + C \quad \longleftarrow \text{ by (3) above}$$

$$= 2\left[\arcsin\left(\frac{t}{2}\right) + \frac{t}{2}\cdot\frac{\sqrt{4-t^2}}{2}\right] + C$$

Continued on reverse.

2. $\boxed{2}$ Find the Partial Fraction Decomposition for

$$\frac{6x^2+4x+3}{x^2(x+1)}.$$

$$\frac{6x^2+4x+3}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$6x^2+4x+3 = Ax(x+1) + B(x+1) + Cx^2$$

Let $x=0$: $3 = B$

$x=-1$: $6-4+3 = C \Rightarrow C=5$

Now equate coefficients to find A

$\underline{x^2}$

$6 = A + C \Rightarrow A = 1$ so $\dfrac{6x^2+4x+3}{x^2(x+1)} = \dfrac{1}{x} + \dfrac{3}{x^2} + \dfrac{5}{x+1}$

3. $\boxed{3}$ Integrate.

$$\int \left(\frac{1}{x+1} + \frac{2}{(x+1)^2} + \frac{x+5}{x^2+4}\right) dx$$

$$= \int \left(\frac{1}{x+1} + \frac{2}{(x+1)^2} + \frac{x}{x^2+4} + \frac{5}{x^2+4} \right) dx$$

$$= \ln|x+1| - \frac{2}{x+1} + \frac{1}{2}\ln(x^2+4) + \frac{5}{2}\arctan\left(\frac{x}{2}\right) + C$$