Math 172 | Quiz 6 | Name: ______________
14 Oct 2016 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{6}$ Find C so that $f(x) = C\left(\frac{x^2+x+1}{(x^2+1)^2}\right)$ is a probability density function on $[0, \infty)$.

HINT: $\frac{x^2+x+1}{(x^2+1)^2} = \frac{x^2+1}{(x^2+1)^2} + \frac{x}{(x^2+1)^2}$

We need to solve $\int_0^\infty f(x)\,dx = 1$.

$$1 = C\int_0^\infty \left(\frac{1}{x^2+1} + \frac{x}{(x^2+1)^2}\right) dx = C \cdot \lim_{R\to\infty}\left[\arctan x - \frac{1}{2}\cdot\frac{1}{x^2+1}\right]\Bigg|_0^R$$

$u = x^2+1$
$du = 2x\,dx$

$$= C \lim_{R\to\infty}\left[\arctan R - \arctan 0 - \frac{1}{2}\cdot\frac{1}{R^2+1} + \frac{1}{2}\right]$$

$$= C\left[\frac{\pi}{2} + \frac{1}{2}\right]$$

so $C = \frac{2}{\pi+1}$.

2. $\boxed{4}$ Use the Comparison Test to show the following converges.

$$\int_{12}^{\infty} \frac{x}{x^{57}+13}\,dx$$

For $x > 0$, $0 \le \frac{x}{x^{57}+13} \le \frac{x}{x^{57}} = \frac{1}{x^{56}}$.

Additionally, $\int_{12}^{\infty} \frac{dx}{x^{56}}$ is a convergent p-integral.

By the Comparison Test, $\int_{12}^{\infty} \frac{x}{x^{57}+13}\,dx$ also converges.